Introduction to two Higgs doublet model

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Outline：

- **Introduction on models**

- **Theoretical and experimental constraints**

T. D. Lee, PRD8 (1973) 1226;
H. E. Haber, G. L. Kane, T. Sterling, NPB161 (1979) 493;
L. J. Hall, M. B. Wise, NPB187 (1981) 397;
J. G. Donoghue, L. F. Li, PRD19 (1979) 945;
V. D. Barger, J. L. Hewett, R. J. N. Phillips, PRD41 (1990) 3421

Two-Higgs-doublet model （2HDM）

The general scalar potential of 2HDM：

$$\begin{aligned} V \;=\;& m_{11}^2(\Phi_1^\dagger\Phi_1) + m_{22}^2(\Phi_2^\dagger\Phi_2) - \left[m_{12}^2(\Phi_1^\dagger\Phi_2 + \text{h.c.})\right] \\ &+\frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ &+\left[\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right] + \left[\lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \text{h.c.}\right] \\ &+\left[\lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \text{h.c.}\right]. \end{aligned}$$

Φ_1, Φ_2 **are complex Higgs doublets with hypercharge Y=1：**

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}\left(v_1 + \phi_1 + i a_1\right) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}\left(v_2 + \phi_2 + i a_2\right) \end{pmatrix}$$

vacuum expectation values $v = \sqrt{v_1^2 + v_2^2} = 246\ \text{GeV}$ $\quad \tan\beta \equiv v_2 / v_1$

Here we focus on the CP-conserving model

The general Yukawa interactions are written as

$$
\begin{aligned}
-\mathcal{L} \;=\;& Y_{u1}\,\overline{Q}_L\,\tilde{\Phi}_1\,u_R \;+\; Y_{u2}\,\overline{Q}_L\,\tilde{\Phi}_2\,u_R \\
&+Y_{d1}\,\overline{Q}_L\,\Phi_1\,d_R + Y_{d2}\,\overline{Q}_L\,\Phi_2\,d_R \\
&+Y_{\ell 1}\,\overline{L}_L\,\Phi_1\,e_R \;+\; Y_{\ell 2}\,\overline{L}_L\,\Phi_2\,e_R + \text{h.c.}\,,
\end{aligned}
$$

$$Q_L^T = (u_L\,, d_L),\; L_L^T = (\nu_L\,, l_L),\; \widetilde{\Phi}_{1,2} = i\tau_2\Phi^*_{1,2}$$

To avoid the tree-level FCNC couplings, we introduce a Z_2 discrete symmetry,

Model	Φ_2	Φ_1	u_R^i	d_R^i	e_R^i
Type I	+	−	+	+	+
Type II	+	−	+	−	−
Lepton-specific	+	−	+	+	−
Flipped	+	−	+	−	+

Four types of 2HDMs without tree-level FCNC

Yukawa interaction under the Z_2 discrete symmetry,

Type-I

$$-\mathcal{L} = Y_{u2}\,\overline{Q}_L\,\tilde{\Phi}_2\,u_R + Y_{d2}\,\overline{Q}_L\,\Phi_2\,d_R + Y_{\ell 2}\,\overline{L}_L\,\Phi_2\,e_R + \text{h.c.}\,.$$

Type-II

$$-\mathcal{L} = Y_{u2}\,\overline{Q}_L\,\tilde{\Phi}_2\,u_R + Y_{d1}\,\overline{Q}_L\,\Phi_1\,d_R + Y_{\ell 1}\,\overline{L}_L\,\Phi_1\,e_R + \text{h.c.}$$

Lepton-specific (Type-X)

$$-\mathcal{L} = Y_{u2}\,\overline{Q}_L\,\tilde{\Phi}_2\,u_R + Y_{d1}\,\overline{Q}_L\,\Phi_2\,d_R + Y_{\ell 1}\,\overline{L}_L\,\Phi_1\,e_R + \text{h.c.}$$

Flipped

$$-\mathcal{L} = Y_{u2}\,\overline{Q}_L\,\tilde{\Phi}_2\,u_R + Y_{d1}\,\overline{Q}_L\,\Phi_1\,d_R + Y_{\ell 1}\,\overline{L}_L\,\Phi_2\,e_R + \text{h.c.}$$

The Higgs potential with a soft Z_2 symmetry,

$$V_{tree} = m_{11}^2(\Phi_1^\dagger\Phi_1) + m_{22}^2(\Phi_2^\dagger\Phi_2) - \left[m_{12}^2(\Phi_1^\dagger\Phi_2 + \text{h.c.})\right] + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \left[\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right].$$

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \phi_1 + ia_1) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \phi_2 + ia_2) \end{pmatrix}$$

m_{11}^2, m_{22}^2 **are determined by the potential minimization condition at**

$$< \phi_1 >= v_1, \quad < \phi_2 >= v_2$$

$$\frac{\partial V_{tree}}{\partial \phi_1} = 0, \quad \frac{\partial V_{tree}}{\partial \phi_2} = 0 \Rightarrow \begin{aligned} m_{11}^2 &= m_{12}^2 t_\beta - \frac{1}{2}v^2\left(\lambda_1 c_\beta^2 + \lambda_{345} s_\beta^2\right) \\ m_{22}^2 &= m_{12}^2/t_\beta - \frac{1}{2}v^2\left(\lambda_2 s_\beta^2 + \lambda_{345} c_\beta^2\right) \end{aligned}$$

$$\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$$

The mass matrix of CP-even Higgses:

$$\begin{pmatrix}\phi_1 & \phi_2\end{pmatrix}\begin{pmatrix} m_{12}^2 t_\beta + \lambda_1 v^2 c_\beta^2 & -m_{12}^2 + \frac{\lambda_{345}}{2} v^2 s_{2\beta} \\ -m_{12}^2 + \frac{\lambda_{345}}{2} v^2 s_{2\beta} & m_{12}^2/t_\beta + \lambda_2 v^2 s_\beta^2 \end{pmatrix}\begin{pmatrix}\phi_1 \\ \phi_2\end{pmatrix}$$

The mass matrix of CP-even Higgses:

$$\begin{pmatrix}a_1 & a_2\end{pmatrix}\left[m_{12}^2 - \frac{1}{2}\lambda_5 v^2 s_{2\beta}\right]\begin{pmatrix} t_\beta & -1 \\ -1 & 1/t_\beta \end{pmatrix}\begin{pmatrix}a_1 \\ a_2\end{pmatrix}$$

The mass matrix of charged Higgses:

$$\begin{pmatrix}\phi_1^+ & \phi_2^+\end{pmatrix}\left[m_{12}^2 - \frac{1}{4}(\lambda_4 + \lambda_5) v^2 s_{2\beta}\right]\begin{pmatrix} t_\beta & -1 \\ -1 & 1/t_\beta \end{pmatrix}\begin{pmatrix}\phi_1^- \\ \phi_2^-\end{pmatrix}$$

The mass eigenstates can be obtained from the original fields by the rotation matrices,

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix},$$

$$\begin{pmatrix} G^0 \\ A \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix},$$

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}.$$

Goldstones: $G^0, \; G^\pm$

Physical states: $h \quad H$ (CP-even) $\quad A$ (CP-odd) $\quad H^\pm$

CP-even CP-odd

The masses of physical states:

$$m_{H,h}^2 = \frac{1}{2}\left[M_{P,11}^2 + M_{P,22}^2 \pm \sqrt{(M_{P,11}^2 - M_{P,22}^2)^2 + 4(M_{P,12}^2)^2}\right]$$

$$m_A^2 = \frac{m_{12}^2}{s_\beta c_\beta} - \lambda_5 v^2 \ ,$$

$$m_{H^\pm}^2 = \frac{m_{12}^2}{s_\beta c_\beta} - \frac{1}{2}(\lambda_4 + \lambda_5) v^2 \ .$$

The gauge-kinetic Lanrangian:

$$\mathcal{L}_{\rm g} = \left(D^\mu \Phi_1\right)^\dagger \left(D_\mu \Phi_1\right) + \left(D^\mu \Phi_2\right)^\dagger \left(D_\mu \Phi_2\right)$$

$$\mathcal{L}_{\rm g} \supset \frac{g^2 + g'^2}{8} v^2 \ ZZ \ \left(1 + 2\frac{h}{v}\sin(\beta - \alpha) + 2\frac{H}{v}\cos(\beta - \alpha)\right)$$
$$+ \frac{g^2}{4} v^2 \ W^+ W^- \ \left(1 + 2\frac{h}{v}\sin(\beta - \alpha) + 2\frac{H}{v}\cos(\beta - \alpha)\right)$$

The fermion mass and Yukawa coupings for type-II 2HDM

$$-\mathcal{L} = Y_{u2}\,\overline{Q}_L\,\tilde{\Phi}_2\,u_R + Y_{d1}\,\overline{Q}_L\,\Phi_1\,d_R + Y_{\ell 1}\,\overline{L}_L\,\Phi_1\,e_R + \text{h.c.}$$

$$\begin{aligned}
-\mathcal{L} = & \frac{vs_\beta}{\sqrt{2}}\ \bar{u}_L Y_{u2} u_R + \frac{1}{\sqrt{2}}(hc_\alpha + Hs_\alpha - iAc_\beta)\ \bar{u}_L Y_{u2} u_R - c_\beta H^-\ \bar{d}_L Y_{u2} u_R + h.c. \\
& + \frac{vc_\beta}{\sqrt{2}}\ \bar{d}_L Y_{d1} d_R + \frac{1}{\sqrt{2}}(-hs_\alpha + Hc_\alpha - iAs_\beta)\ \bar{d}_L Y_{d1} d_R - s_\beta H^+\ \bar{u}_L Y_{d1} d_R + h.c. \\
& + \frac{vc_\beta}{\sqrt{2}}\ \bar{\ell}_L Y_{\ell 1} e_R + \frac{1}{\sqrt{2}}(-hs_\alpha + Hc_\alpha - iAs_\beta)\ \bar{\ell}_L Y_{\ell 1} e_R - s_\beta H^+\ \bar{\nu}_L Y_{\ell 1} e_R + h.c.
\end{aligned}$$

Rotating the interaction eigenstates to mass eigenstates：

$$u_L^m = V_{uL} u_L\ , u_R^m = V_{uR} u_R\ , d_L^m = V_{dL} d_L\ , d_R^m = V_{dR} d_R, V_{CKM} \equiv V_{uL} V_{dL}^\dagger$$

$$V_{uL} Y_{u2} V_{uR}^\dagger = diag(\frac{\sqrt{2}m_t}{vs_\beta}, \frac{\sqrt{2}m_c}{vs_\beta}, \frac{\sqrt{2}m_u}{vs_\beta})$$

$$V_{dL} Y_{d1} V_{dR}^\dagger = diag(\frac{\sqrt{2}m_b}{vc_\beta}, \frac{\sqrt{2}m_s}{vc_\beta}, \frac{\sqrt{2}m_d}{vc_\beta}).$$

We obtain the fermion mass and their couplings,

$$
\begin{aligned}
-\mathcal{L} = & \, m_u \bar{u}_L u_R + \frac{m_u}{v s_\beta}(h c_\alpha + H s_\alpha - i A c_\beta) \; \bar{u}_L u_R + h.c. \\
& + m_d \bar{d}_L d_R + \frac{m_d}{v c_\beta}(-h s_\alpha + H c_\alpha - i A s_\beta) \; \bar{d}_L d_R + h.c. \\
& - H^+ \; (\frac{\sqrt{2} m_d}{v} t_\beta \bar{u}_L \; V_{CKM} \; d_R + \frac{\sqrt{2} m_u}{v t_\beta} \bar{u}_R \; V_{CKM} \; d_L) + h.c. \\
& + m_\ell \bar{\ell}_L e_R + \frac{m_\ell}{v c_\beta}(-h s_\alpha + H c_\alpha - i A s_\beta) \; \bar{\ell}_L e_R - \frac{\sqrt{2} m_\ell}{v} t_\beta H^+ \; \bar{\nu}_L e_R + h.c.
\end{aligned}
$$

$$
\begin{aligned}
\frac{c_\alpha}{s_\beta} &= \sin(\beta - \alpha) + \cos(\beta - \alpha) \frac{1}{t_\beta} \\
\frac{s_\alpha}{s_\beta} &= \cos(\beta - \alpha) - \sin(\beta - \alpha) \frac{1}{t_\beta} \\
-\frac{s_\alpha}{c_\beta} &= \sin(\beta - \alpha) + \cos(\beta - \alpha)(-t_\beta) \\
\frac{c_\alpha}{c_\beta} &= \cos(\beta - \alpha) - \sin(\beta - \alpha)(-t_\beta)
\end{aligned}
$$

The Yukawa couplings can be expressed as,

$$
\begin{aligned}
-\mathcal{L}_Y = & \frac{m_f}{v} \left(\sin(\beta-\alpha) + \cos(\beta-\alpha)\kappa_f\right) h\bar{f}f \\
& + \frac{m_f}{v} \left(\cos(\beta-\alpha) - \sin(\beta-\alpha)\kappa_f\right) H\bar{f}f \\
& - i\frac{m_u}{v}\kappa_u \, A\bar{u}\gamma_5 u + i\frac{m_d}{v}\kappa_d \, A\bar{d}\gamma_5 d + i\frac{m_\ell}{v}\kappa_\ell \, A\bar{\ell}\gamma_5 \ell \\
& + H^+ \, \bar{u} \, V_{CKM} \, \left(\frac{\sqrt{2}m_d}{v}\kappa_d P_R - \frac{\sqrt{2}m_u}{v}\kappa_u P_L\right)d + h.c. \\
& + \frac{\sqrt{2}m_\ell}{v}\kappa_\ell H^+ \, \bar{\nu} P_R e + h.c.
\end{aligned}
$$

	I	II	lepton-specific	flipped
κ_u	$1/t_\beta$	$1/t_\beta$	$1/t_\beta$	$1/t_\beta$
κ_d	$1/t_\beta$	$-t_\beta$	$1/t_\beta$	$-t_\beta$
κ_ℓ	$1/t_\beta$	$-t_\beta$	$-t_\beta$	$1/t_\beta$

Inert Higgs-doublet-model

N. G. Deshpande, E. Ma, PRD18 (1978) 2574

An exact *Z2* discrete symmetry is imposed, under which all SM fields are taken to be even, while the new (inert) doublet is odd.

$$\mathcal{V} = m_{11}^2(\Phi_1^\dagger\Phi_1) + m_{22}^2(\Phi_2^\dagger\Phi_2) + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \left[\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right]$$

$$\Phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v+h+iG) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H+iA) \end{pmatrix}$$

Φ_2 has no VeV. m_{11}^2 is determined by requiring the scalar potential minimization condition <h>=v,

$$m_{11}^2 = -\frac{1}{2}\lambda_1 v^2$$

The masses of physical states:

$$m_{H^\pm}^2 = m_{22}^2 + \frac{\lambda_3}{2} v^2, \quad m_A^2 = m_{H^\pm}^2 + \frac{1}{2}(\lambda_4 - \lambda_5) v^2.$$

$$m_h^2 = \lambda_1 v^2 \equiv (125 \text{ GeV})^2, \quad m_H^2 = m_A^2 + \lambda_5 v^2.$$

The fermion masses are given via the Yukawa interaction with Φ_1,

$$-\mathcal{L} = y_u \overline{Q}_L \tilde{\Phi}_1 u_R + y_d \overline{Q}_L \Phi_1 d_R + y_l \overline{L}_L \Phi_1 e_R + \text{h.c.}\,,$$

Because of the exact Z_2 symmetry, either of the lightest neutral component *H* and *A* is stable and may be considered as a DM candiate.

Higgs Basis

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \phi_1 + ia_1) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \phi_2 + ia_2) \end{pmatrix}$$

$$v^2 = v_1^2 + v_2^2, \quad t_\beta = \frac{v_2}{v_1}$$

Redefining Higgs doublets as,

$$H_1 = \begin{pmatrix} G^+ \\ \frac{h_1 + v + iG}{\sqrt{2}} \end{pmatrix} = \Phi_1 c_\beta + \Phi_2 s_\beta \qquad H_2 = \begin{pmatrix} H^+ \\ \frac{h_2 + iA}{\sqrt{2}} \end{pmatrix} = -\Phi_1 s_\beta + \Phi_2 c_\beta$$

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix},$$

$$= \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix},$$

$$= \begin{pmatrix} \cos(\beta-\alpha) & \sin(\beta-\alpha) \\ -\sin(\beta-\alpha) & \cos(\beta-\alpha) \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}.$$

In Higgs basis，*H_2* has no VeV，and there is no mixing between $G^\pm$ and $H^\pm$, G^0 and A

$$
\begin{aligned}
V_{tree} &= m_{11}^2(\Phi_1^\dagger\Phi_1) + m_{22}^2(\Phi_2^\dagger\Phi_2) - \left[m_{12}^2(\Phi_1^\dagger\Phi_2 + \text{h.c.})\right] \\
&+ \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\
&+ \left[\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right].
\end{aligned}
$$

The potential, when expressed in terms of *H_1* and *H_2*, has the form as

$$
\begin{aligned}
\mathcal{V} &= Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + Y_3[H_1^\dagger H_2 + \text{h.c.}] + \tfrac{1}{2}Z_1(H_1^\dagger H_1)^2 + \tfrac{1}{2}Z_2(H_2^\dagger H_2)^2 + Z_3(H_1^\dagger H_1)(H_2^\dagger H_2) \\
&+ Z_4(H_1^\dagger H_2)(H_2^\dagger H_1) + \left\{\tfrac{1}{2}Z_5(H_1^\dagger H_2)^2 + \left[Z_6(H_1^\dagger H_1) + Z_7(H_2^\dagger H_2)\right]H_1^\dagger H_2 + \text{h.c.}\right\},
\end{aligned}
$$

$$
\begin{aligned}
Y_1 &= m_{11}^2 c_\beta^2 + m_{22}^2 s_\beta^2 - m_{12}^2 s_{2\beta}, \\
Y_2 &= m_{11}^2 s_\beta^2 + m_{22}^2 c_\beta^2 + m_{12}^2 s_{2\beta}, \\
Y_3 &= \tfrac{1}{2}(m_{22}^2 - m_{11}^2)s_{2\beta} - m_{12}^2 c_{2\beta}.
\end{aligned}
$$

$$
\begin{aligned}
Z_1 &\equiv \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \tfrac{1}{2}\lambda_{345}s_{2\beta}^2, \\
Z_2 &\equiv \lambda_1 s_\beta^4 + \lambda_2 c_\beta^4 + \tfrac{1}{2}\lambda_{345}s_{2\beta}^2, \\
Z_i &\equiv \tfrac{1}{4}s_{2\beta}^2\left[\lambda_1 + \lambda_2 - 2\lambda_{345}\right] + \lambda_i, \\
Z_6 &\equiv -\tfrac{1}{2}s_{2\beta}\left[\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - \lambda_{345}c_{2\beta}\right] \\
Z_7 &\equiv -\tfrac{1}{2}s_{2\beta}\left[\lambda_1 s_\beta^2 - \lambda_2 c_\beta^2 + \lambda_{345}c_{2\beta}\right]
\end{aligned}
$$

S. Davidson, H. E. Haber, PRD72 (2005) 035004;

J. Bernon, J. F. Gunion, H. E. Haber, Y. Jiang, S. Kraml, PRD92 (2015) 075004

Aligned 2HDM in the Higgs basis

A. Pich, P. Tuzon, PRD80 (2009) 091702

Y_1 and Y_3 are determined by the scalar potential minimum condition at

$$< h_1 >= v \qquad < h_2 >= 0$$

$$\frac{\partial \mathcal{V}}{\partial h_1} = 0, \qquad \frac{\partial \mathcal{V}}{\partial h_2} = 0 \quad \Rightarrow \quad \begin{aligned} Y_1 &= -\tfrac{1}{2} Z_1 v^2 \\ Y_3 &= -\tfrac{1}{2} Z_6 v^2 \end{aligned}$$

The masses of charged Higgs and CP-odd Higgs:

$$\begin{aligned} m_{H^\pm}^2 &= Y_2 + \tfrac{1}{2} Z_3 v^2 \,, \\ m_A^2 &= Y_2 + \tfrac{1}{2}(Z_3 + Z_4 - Z_5) v^2 \end{aligned}$$

The mass matrix of CP-even Higgses:

$$\begin{pmatrix} Z_1 v^2 & Z_6 v^2 \\ Z_6 v^2 & m_A^2 + Z_5 v^2 \end{pmatrix} \quad \Rightarrow$$

$$m_{H,h}^2 = \tfrac{1}{2}\left[m_A^2 + (Z_1 + Z_5)v^2 \pm \Delta_H\right],$$

$$\Delta_H \equiv \sqrt{\left[m_A^2 + (Z_5 - Z_1)v^2\right]^2 + 4 Z_6^2 v^4}$$

$$\tan 2\theta \equiv \tan 2(\beta - \alpha) = \frac{2 Z_6 v^2}{m_A^2 + (Z_5 - Z_1) v^2}$$

In the aligned 2HDM, the Yukawa interaction without tree-level FCNC

$$-\mathcal{L} = y_u \overline{Q}_L (\tilde{\Phi}_1 + \kappa_u \tilde{\Phi}_2) u_R + y_d \overline{Q}_L (\Phi_1 + \kappa_d \Phi_2) d_R + y_l \overline{L}_L (\Phi_1 + \kappa_\ell \Phi_2) e_R + \text{h.c.}\,,$$

We can obtain the Yukawa coupling

$$\begin{aligned} -\mathcal{L}_Y = & \frac{m_f}{v} \left(\sin\theta + \cos\theta \kappa_f\right) h\bar{f}f \\ & + \frac{m_f}{v} \left(\cos\theta - \sin\theta \kappa_f\right) H\bar{f}f \\ & - i\frac{m_u}{v}\kappa_u \; A\bar{u}\gamma_5 u + i\frac{m_d}{v}\kappa_d \; A\bar{d}\gamma_5 d + i\frac{m_\ell}{v}\kappa_\ell \; A\bar{\ell}\gamma_5 \ell \\ & + H^+ \; \bar{u} \, V_{CKM} \left(\frac{\sqrt{2}m_d}{v}\kappa_d P_R - \frac{\sqrt{2}m_u}{v}\kappa_u P_L\right) d + h.c. \\ & + \frac{\sqrt{2}m_\ell}{v}\kappa_\ell H^+ \; \bar{\nu} P_R e + h.c. \end{aligned}$$

General 2HDM in the Higgs basis

The general Yukawa intactions,

$$
\begin{aligned}
-\mathcal{L} \;=\; & Y_{u1}\,\overline{Q}_L\,\tilde{H}_1\,u_R \;+\; Y_{u2}\,\overline{Q}_L\,\tilde{H}_2\,u_R \\
& +Y_{d1}\,\overline{Q}_L\,H_1\,d_R \;+\; Y_{d2}\,\overline{Q}_L\,H_2\,d_R \\
& +Y_{\ell 1}\,\overline{L}_L\,H_1\,e_R \;+\; Y_{\ell 2}\,\overline{L}_L\,H_2\,e_R \;+\; \text{h.c.}\,,
\end{aligned}
$$

$$
H_1 = \begin{pmatrix} G^+ \\ \frac{h_1+v+iG}{\sqrt{2}} \end{pmatrix} \qquad H_2 = \begin{pmatrix} H^+ \\ \frac{h_2+iA}{\sqrt{2}} \end{pmatrix}
$$

Rotating the interaction eigenstates to mass eigenstates：

$$
u_L^m = V_{uL}u_L \;, u_R^m = V_{uR}u_R \;, d_L^m = V_{dL}d_L \;, d_R^m = V_{dR}d_R, V_{CKM} \equiv V_{uL}V_{dL}^\dagger
$$

$$
V_{uL}Y_{u1}V_{uR}^\dagger = diag(\frac{\sqrt{2}m_t}{v}, \frac{\sqrt{2}m_c}{v}, \frac{\sqrt{2}m_u}{v})
$$

$$
V_{dL}Y_{d1}V_{dR}^\dagger = diag(\frac{\sqrt{2}m_b}{v}, \frac{\sqrt{2}m_s}{v}, \frac{\sqrt{2}m_d}{v})
$$

$$
V_{uL}Y_{u2}V_{uR}^\dagger = X_{u2}
$$

$$
V_{dL}Y_{d2}V_{dR}^\dagger = X_{d2}
$$

$$
Y_{\ell 1} = diag(\frac{\sqrt{2}m_e}{v}, \frac{\sqrt{2}m_\mu}{v}, \frac{\sqrt{2}m_\tau}{v})
$$

$$
\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}
$$

The general Yukawa coupling

$$
\begin{aligned}
-\mathcal{L} = {} & m_{u_i}\bar{u}_i u_i + m_{d_i}\bar{d}_i d_i + m_{\ell_i}\bar{\ell}_i \ell_i \\
& + \left[\frac{m_{qi}}{v}\sin\theta\delta_{ij} + \cos\theta\frac{X_{q2}^{ij}}{\sqrt{2}}\right] \; h\bar{q}_{iL}q_{jR} + h.c. \\
& + \left[\frac{m_{qi}}{v}\cos\theta\delta_{ij} - \sin\theta\frac{X_{q2}^{ij}}{\sqrt{2}}\right] \; H\bar{q}_{iL}q_{jR} + h.c. \\
& + \left[\frac{m_{\ell i}}{v}\sin\theta\delta_{ij} + \cos\theta\frac{Y_{\ell 2}^{ij}}{\sqrt{2}}\right] \; h\bar{\ell}_{iL}\ell_{jR} + h.c. \\
& + \left[\frac{m_{\ell i}}{v}\cos\theta\delta_{ij} - \sin\theta\frac{Y_{\ell 2}^{ij}}{\sqrt{2}}\right] \; H\bar{\ell}_{iL}\ell_{jR} + h.c. \\
& - i\frac{X_{u2}^{ij}}{\sqrt{2}}A\bar{u}_{iL}u_{jR} + i\frac{X_{d2}^{ij}}{\sqrt{2}}A\bar{d}_{iL}d_{jR} + i\frac{Y_{\ell 2}^{ij}}{\sqrt{2}}A\bar{\ell}_{iL}\ell_{jR} + h.c. \\
& + H^+ \; \left[\bar{u}_{iL} \; (V_{CKM}X_{d2})^{ij} \; d_{jR} - \bar{u}_{iR} \; \left(X_{u2}^\dagger V_{CKM}\right)^{ij} \; d_{jL}\right] + h.c. \\
& + Y_{\ell 2}^{ij}H^+ \; \bar{\nu}_{iL}\ell_{jR} + h.c.
\end{aligned}
$$

Cheng-Sher Ansatz: $X_{ij} = \frac{1}{v}\sqrt{2m_i m_j}\lambda_{ij}$, λ_{ij} are of order one

The ansatz weakens the bound on FCNC from the first two generations.

T. P. Cheng, M. Sher, PRD35 (1987) 3484

Vacuum stability

$$V_{tree} = m_{11}^2(\Phi_1^\dagger\Phi_1) + m_{22}^2(\Phi_2^\dagger\Phi_2) - \left[m_{12}^2(\Phi_1^\dagger\Phi_2 + \text{h.c.})\right]$$
$$+\frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$
$$+\left[\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right].$$

We parameterise the fields

$$\Phi_1^\dagger\Phi_1 = X_1^2, \quad \Phi_2^\dagger\Phi_2 = X_2^2, \quad \Phi_1^\dagger\Phi_2 = X_1X_2\rho e^{i\theta} \text{ with } 0 \le \rho \le 1.$$

The quartic parts

$$V_4 = \frac{\lambda_1}{2}X_1^4 + \frac{\lambda_2}{2}X_2^4 + \lambda_3X_1^2X_2^2 + \lambda_4X_1^2X_2^2\rho^2 + \lambda_5X_1^2X_2^2\rho^2\cos 2\theta.$$

After stabilizing θ at the minimum, we obtain the θ independent part

$$V_{\theta-indep} = \frac{\lambda_1}{2}X_1^4 + \frac{\lambda_2}{2}X_2^4 + \lambda_3X_1^2X_2^2 + \lambda_4X_1^2X_2^2\rho^2 - |\lambda_5| X_1^2X_2^2\rho^2.$$

For $\lambda_4 - |\lambda_5| > 0$, the potential has minimal value at $\rho = 0$,

$$V_{\theta-\rho-indep} = \frac{\lambda_1}{2}X_1^4 + \frac{\lambda_2}{2}X_2^4 + \lambda_3 X_1^2 X_2^2$$
$$= \left(\sqrt{\frac{\lambda_1}{2}}X_1^2 - \sqrt{\frac{\lambda_2}{2}}X_2^2\right)^2 + \lambda_3 X_1^2 X_2^2 + \sqrt{\lambda_1\lambda_2}X_1^2 X_2^2.$$

For $\lambda_4 - |\lambda_5| < 0$, the potential has minimal value at $\rho = 1$,

$$V_{\theta-\rho-indep} = \frac{\lambda_1}{2}X_1^4 + \frac{\lambda_2}{2}X_2^4 + \lambda_3 X_1^2 X_2^2 + \lambda_4 \mid X_1^2 X_2^2 - \mid \lambda_5 \mid X_1^2 X_2^2.$$

The vacuum stability requires

$$\lambda_1 \geq 0, \quad \lambda_2 \geq 0, \quad \lambda_3 + \sqrt{\lambda_1\lambda_2} \geq 0, \quad \lambda_3 + \lambda_4 - \mid \lambda_5 \mid + \sqrt{\lambda_1\lambda_2} \geq 0$$

Applying the criteria of arXiv:1205.3781 (Kristjan Kannike, EPJC72 (2012) 2093)

The matrix of quartic couplings for the potential in the $(X_1^2, \ X_2^2)$ basis

$$\begin{pmatrix} \frac{\lambda_1}{2} & \frac{\lambda_3+(\lambda_4-|\lambda_5|)\rho^2}{2} \\ \frac{\lambda_3+(\lambda_4-|\lambda_5|)\rho^2}{2} & \frac{\lambda_2}{2} \end{pmatrix}$$

A symmetric matrix of order 2:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix} \Rightarrow \begin{array}{c} a_{11} \geqslant 0, a_{22} \geqslant 0, \\ a_{12} + \sqrt{a_{11}a_{22}} \geqslant 0. \end{array}$$

A symmetric matrix of order 3, tha vaccum stability requires

$$a_{11} \geqslant 0, a_{22} \geqslant 0, a_{33} \geqslant 0,$$

$$\bar{a}_{12} = a_{12} + \sqrt{a_{11}a_{22}} \geqslant 0,$$

$$\bar{a}_{13} = a_{13} + \sqrt{a_{11}a_{33}} \geqslant 0,$$

$$\bar{a}_{23} = a_{23} + \sqrt{a_{22}a_{33}} \geqslant 0,$$

$$\sqrt{a_{11}a_{22}a_{33}} + a_{12}\sqrt{a_{33}} + a_{13}\sqrt{a_{22}} + a_{23}\sqrt{a_{11}} + \sqrt{2\bar{a}_{12}\bar{a}_{13}\bar{a}_{23}} \geqslant 0.$$

Unitarity

B. W. Lee, C. Quigg, H. B. Thacker, PRD16 (1977) 5;
M. D. Goodsell, F. Staub, EPJC78 (2018) .

The *S* matrix in terms of the interaction matrix *T*

$$S = 1 + iT$$

$$T_{ba} \equiv {}_{\text{out}}\langle\{k,b\}|iT|\{p,a\}\rangle_{\text{in}} \equiv i\mathcal{M}(\{p,a\} \to \{k,b\})(2\pi)^4\delta^4(\{k\}-\{p\}) \equiv i\mathcal{M}_{ba}(2\pi)^4\delta^4(\{k\}-\{p\}),$$

$$SS^\dagger = 1 \longrightarrow T^\dagger T + i(T - T^\dagger) = TT^\dagger + i(T - T^\dagger) = 0.$$

We insert a complete set of states

$$\langle\{k,b\}|T^\dagger T|\{p,a\}\rangle = \sum_n d\Pi_n \langle\{k,b\}|T^\dagger|\{q_n,c_n\}\rangle\langle\{q_n,c_n\}|T|\{p,a\}\rangle.$$

$$-i(\mathcal{M}_{ba}^{2\to2} - (\mathcal{M}_{ba}^{2\to2})^\dagger) = \sum_c \frac{1}{2^{\delta_c}}\frac{|\mathbf{p}_c|}{16\pi^2\sqrt{s}}\int d\Omega \mathcal{M}_{ca}^{2\to2}\mathcal{M}_{cb}^{*2\to2} + \underbrace{\sum_{n\neq2} d\Pi_n \mathcal{M}_{ca}^{2\to n}\mathcal{M}_{cb}^{*2\to n}}_{\geq0}.$$

We define p_a, k_b, p_c to lie along the unit vectors

$$\hat{k}_a = (1,0,0)$$

$$\hat{k}_b = (z_b, \sin\theta_b, 0)$$

$$\hat{k}_c = (z_c, \sin\theta_c\cos\phi_c, \sin\theta_c\sin\phi_c), \qquad z_b \equiv \cos\theta_b, \qquad z_c \equiv \cos\theta_c.$$

We decompose the matrices into partial waves:

$$\mathcal{M}_{ba} = 16\pi \sum (2J+1)\hat{a}_J(s) P_J(z_b)$$

$$\mathcal{M}_{ca} = 16\pi \sum (2J+1)\hat{a}_J(s) P_J(z_c)$$

$$\mathcal{M}_{cb} = 16\pi \sum (2J+1)\hat{a}_J(s) P_J(\hat{k}_b \cdot \hat{k}_c),$$

Apply the relation $\displaystyle \int_{-1}^{1} dz P_J(z) P_{J'}(z) = \frac{2}{2J+1}\delta_{JJ'}, \qquad P_0(z) = 1,$

$$-2\pi i(\hat{a}^J - \hat{a}^{J\dagger})_{ba} \geq \sum_c \frac{2^{-\delta_c}|\mathbf{p}_c|}{\sqrt{s}}(2J'+1)(2J''+1)\int d\phi_c dz_c dz_b P_J(z_b) P_{J'}(z_c) P_{J''}(\hat{k}_b \cdot \hat{k}_c)\hat{a}^{J'}_{ca}\hat{a}^{*J''}_{cb}.$$

$$P_{J''}(\hat{k}_b \cdot \hat{k}_c) = \frac{4\pi}{2J''+1}\sum_{m=-J''}^{J} Y_{J''m}(\theta_b,\phi_b) Y^*_{J''m}(\theta_c,\phi_c)$$

$$+\frac{4\pi}{2J''+1}\left[\frac{2J''+1}{4\pi}P_{J''}(z_b)P_{J''}(z_c) + \sum_{m\neq 0} Y_{J''m}(\theta_b,\phi_b) Y^*_{J''m}(\theta_c,\phi_c)\right]$$

$$+\frac{4\pi}{2J''+1}\left[\frac{2J''+1}{4\pi}P_{J''}(z_b)P_{J''}(z_c) + \sum_{m\neq 0} \propto P^m_{J''}(z_b) e^{im\phi_c} Y^*_{J''m}(z_c)\right]$$

Here $\displaystyle Y_{Jm} \propto e^{im\phi} P^m_J(\cos\theta), \qquad Y_{J0} = \sqrt{\frac{2J+1}{4\pi}} P_J(\cos\theta), \phi_b = 0.$

$$-2\pi i(\hat{a}^J - \hat{a}^{J\dagger})_{ba} \geq \sum_c \frac{2^{-\delta_c}|\mathbf{p}_c|}{\sqrt{s}}(2J'+1)(2J''+1)2\pi \int dz_c dz_b P_J(z_b)P_{J'}(z_c)P_{J''}(z_b)P_{J''}(z_c)\hat{a}^{J'}_{ca}\hat{a}^{*J''}_{cb}$$

$$\Rightarrow \quad -2\pi i(\hat{a}^J - \hat{a}^{J\dagger})_{ba} \geq \sum_c \frac{2^{-\delta_c}|\mathbf{p}_c|}{\sqrt{s}} 8\pi$$

$$-\frac{i}{2}(\hat{a}^J - \hat{a}^{J\dagger})_{ba} \geq \sum_c \frac{2^{-\delta_c}|2\mathbf{p}_c|}{\sqrt{s}}\hat{a}^J_{ca}\hat{a}^{J*}_{cb}.$$

Making the definition $a^{ba}_J \equiv \sqrt{\frac{4|\mathbf{p}_b||\mathbf{p}_a|}{2^{\delta_a}2^{\delta_b}s}}\hat{a}^{ba}_J.$

$$\Rightarrow \quad -\frac{i}{2}(a^J - a^{J\dagger})_{ba} \geq \sum_c a^J_{ca}a^{J*}_{cb}$$

We diagonalize a, $a^\dagger$ **, and the eigenvalues satisfy** $\mathrm{Im}(a^i_J) \geq |a^i_J|^2$

$$\Rightarrow \quad [(\mathrm{Re}(a^i_J)]^2 + \left[(\mathrm{Im}a^i_J) - \frac{1}{2}\right]^2 \leq \frac{1}{4}$$

At the tree-level, the bound can be relaxed to

$$|\ \mathrm{Re}(a^i_J)\ | \leq \frac{1}{2}$$

We assume the external masses are vanishing at the high-energy limit, and focus on the J=0 partial wave. The modified zeroth partital waves for $S_1S_2 \to S_3S_4$

$$a_0 \simeq \frac{1}{16\pi}\left(2^{-\frac{1}{2}(\delta_{12}+\delta_{34})}Q_{1234}\right).$$

Q_{1234} is quartic coupling of $S_1S_2S_3S_4$

For the 2HDM, one can take the uncoupled sets of scalar pairs

$$\left\{\phi_1^+\phi_2^-, \phi_1^-\phi_2^+, \phi_1\phi_2, \phi_1 a_2, a_1\phi_2, a_1a_2\right\},$$

$$\left\{\phi_1^+\phi_1, \phi_1^+a_1, \phi_2^+\phi_2, \phi_2^+a_2\right\},$$

$$\left\{\phi_1^+\phi_2, \phi_1^+a_2, \phi_2^+\phi_1, \phi_2^+a_1\right\},$$

$$\left\{\phi_1a_1, \phi_2a_2\right\},$$

$$\left\{\phi_1^+\phi_1^-, \phi_2^+\phi_2^-, \phi_1\phi_1, \phi_2\phi_2, a_1a_1, a_2a_2\right\}$$

We can obtain different eigenvalues

$$
\begin{aligned}
a_\pm &= \tfrac{3}{2}(\lambda_1+\lambda_2) \pm \sqrt{\tfrac{9}{4}(\lambda_1-\lambda_2)^2+(2\lambda_3+\lambda_4)^2}\,, \\
b_\pm &= \tfrac{1}{2}(\lambda_1+\lambda_2) \pm \sqrt{\tfrac{1}{4}(\lambda_1-\lambda_2)^2+\lambda_4^2}\,, \\
c_\pm &= \tfrac{1}{2}(\lambda_1+\lambda_2) \pm \sqrt{\tfrac{1}{4}(\lambda_1-\lambda_2)^2+\lambda_5^2}\,, \\
\mathsf{e}_\pm &= \lambda_3+2\lambda_4 \pm 3\lambda_5\,, \\
\mathtt{f}_\pm &= \lambda_3 \pm \lambda_4\,, \\
\mathsf{g}_\pm &= \lambda_3 \pm \lambda_5\,,
\end{aligned}
$$

The unitarity requirement for the $S_1S_2 \rightarrow S_3S_4$ **amplitudes then translates into the constraints**

$$
|a_\pm|, |b_\pm|, |c_\pm|, |\mathsf{e}_\pm|, |\mathtt{f}_\pm|, |\mathsf{g}_\pm| \leq 8\pi\,.
$$

A. G. Akerod, A. Arhrib, E. M. Naimi, PLB490 (2000) 119;
S. Kanemura, T. Kubota, E. TAkasugi, PLB313 (1993) 155

Signal data of 125 GeV Higgs

The neutral Higgs couplings with gauge boson normalized to SM

$$y_h^V = \sin(\beta - \alpha), \quad y_H^V = \cos(\beta - \alpha),$$

The neutral Higgs Yukawa couplings normalized to SM

$$y_h^{f_i} = \left[\sin(\beta - \alpha) + \cos(\beta - \alpha)\kappa_f\right],$$

$$y_H^{f_i} = \left[\cos(\beta - \alpha) - \sin(\beta - \alpha)\kappa_f\right],$$

Taking the light CP-even Higgs as the 125 GeV Higgs, and the signal data of 125 GeV Higgs allow *h* to have different two types of couplings

$$y_h^{f_i} \times y_h^V > 0 \text{ for SM} - \text{like coupling},$$

$$y_h^{f_i} \times y_h^V < 0 \text{ for wrong sign Yukawa coupling}.$$

Wrong sign Yukawa coupling

Tha absolute values of $y_h^{f_i}$ and y_h^V should be closed to 1

$$y_h^{f_i} = -1 + \epsilon, \quad y_h^V \simeq 1 - 0.5\cos^2(\beta-\alpha) \quad \text{for} \quad \sin(\beta-\alpha) > 0 \text{ and } \cos(\beta-\alpha) > 0 \ ,$$

$$y_h^{f_i} = 1 - \epsilon, \quad y_h^V \simeq -1 + 0.5\cos^2(\beta-\alpha) \quad \text{for} \quad \sin(\beta-\alpha) < 0 \text{ and } \cos(\beta-\alpha) > 0.$$

The wrong sign Yukawa coupling favors

$$\kappa_f = \frac{-2 + \varepsilon + 0.5\cos(\beta-\alpha)^2}{\cos(\beta-\alpha)} << -1 \text{ for } \sin(\beta-\alpha) > 0 \text{ and } \cos(\beta-\alpha) > 0 \ ,$$

$$\kappa_f = \frac{2 - \varepsilon - 0.5\cos(\beta-\alpha)^2}{\cos(\beta-\alpha)} >> 1 \text{ for } \sin(\beta-\alpha) < 0 \text{ and } \cos(\beta-\alpha) > 0 \ .$$

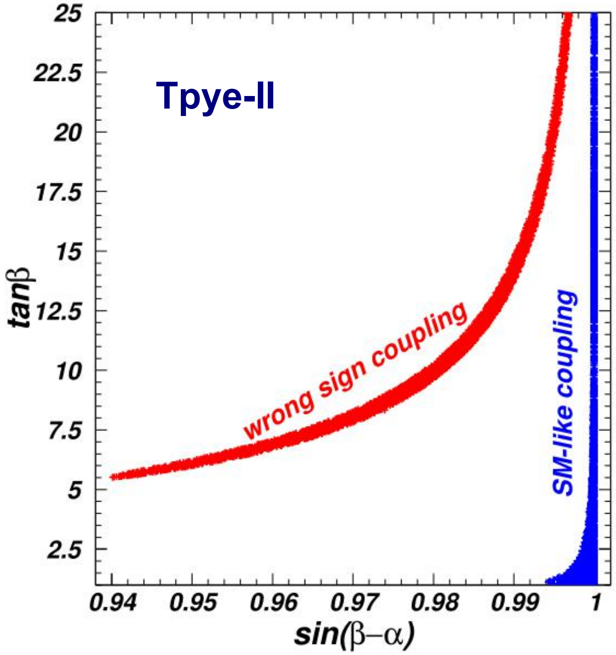
	I	II	lepton-specific	flipped
κ_u	$1/t_\beta$	$1/t_\beta$	$1/t_\beta$	$1/t_\beta$
κ_d	$1/t_\beta$	$-t_\beta$ ✔	$1/t_\beta$	$-t_\beta$ ✔
κ_ℓ	$1/t_\beta$	$-t_\beta$ ✔	$-t_\beta$ ✔	$1/t_\beta$

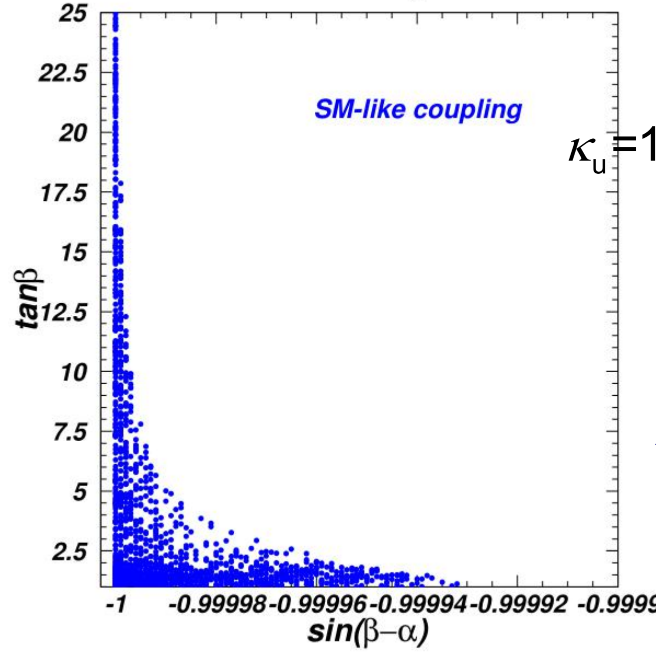
$$H^+ \ \bar{u}\left(\frac{\sqrt{2}m_d}{v}\kappa_d P_R - \frac{\sqrt{2}m_u}{v}\kappa_u P_L\right)d + h.c.$$

$t_\beta \ll 1$ is excluded by the $b \to s\gamma$

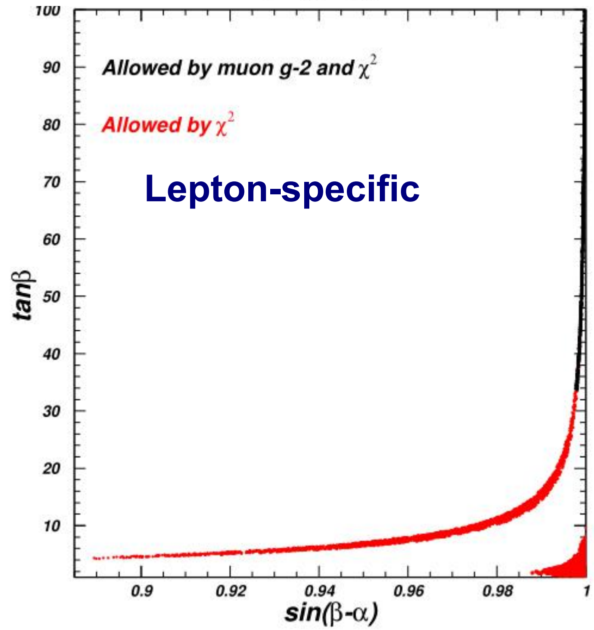
$$y_f^h = [\sin(\beta-\alpha) + \cos(\beta-\alpha)\kappa_f],$$
$$y_V^h = \sin(\beta-\alpha),$$

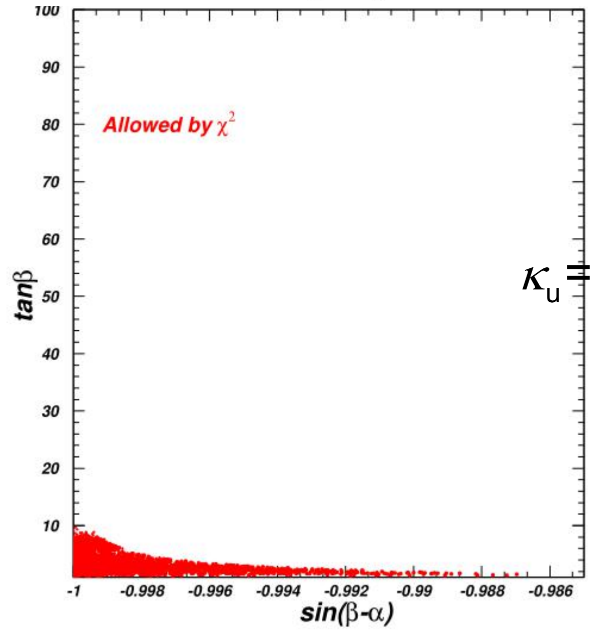
$\kappa_u = 1/\tan\beta, \quad \kappa_\ell = \kappa_d = -\tan\beta,$





X.-F. Han, L W, Y. Zhang, PRD103 (2021) 035012





L W, X.-F. Han, JHEP05 (2015) 039

$\kappa_u = \kappa_d = 1/\tan\beta, \quad \kappa_\ell = -\tan\beta,$

Oblique parameter S, T, U

The 2HDM gives additional contributes to S, T, U via the gauge-boson self-energies diagrams with loops of $h, H, A, H^{\pm}$

The S, T, U parameters can impose strong constraints on the mass specturm of $H, A, H^{\pm}$. One of *H* and *A* is favored to have small mass splitting from $H^{\pm}$.

$$T = \frac{1}{16\pi M_W^2 s_W^2}\Big\{\sin^2(\beta-\alpha)\Big[\mathcal{F}(M_{H^\pm}^2, M_H^2) - \mathcal{F}(M_H^2, M_A^2) + 3\,\mathcal{F}(M_Z^2, M_h^2) - 3\,\mathcal{F}(M_W^2, M_h^2)\Big]$$

$$+ \cos^2(\beta-\alpha)\Big[\mathcal{F}(M_{H^\pm}^2, M_h^2) - \mathcal{F}(M_h^2, M_A^2) + 3\,\mathcal{F}(M_Z^2, M_H^2) - 3\,\mathcal{F}(M_W^2, M_H^2)\Big]$$

$$+ \mathcal{F}(M_{H^\pm}^2, M_A^2) - 3\,\mathcal{F}(M_Z^2, M_{h,\mathrm{ref}}^2) + 3\,\mathcal{F}(M_W^2, M_{h,\mathrm{ref}}^2)\Big\},$$

$$S = \frac{1}{\pi M_Z^2}\Big\{\sin^2(\beta-\alpha)\Big[\mathcal{B}_{22}(M_Z^2; M_Z^2, M_h^2) - M_Z^2\,\mathcal{B}_0(M_Z^2; M_Z^2, M_h^2) + \mathcal{B}_{22}(M_Z^2; M_H^2, M_A^2)\Big]$$

$$+ \cos^2(\beta-\alpha)\Big[\mathcal{B}_{22}(M_Z^2; M_Z^2, M_H^2) - M_Z^2\,\mathcal{B}_0(M_Z^2; M_Z^2, M_H^2) + \mathcal{B}_{22}(M_Z^2; M_h^2, M_A^2)\Big]$$

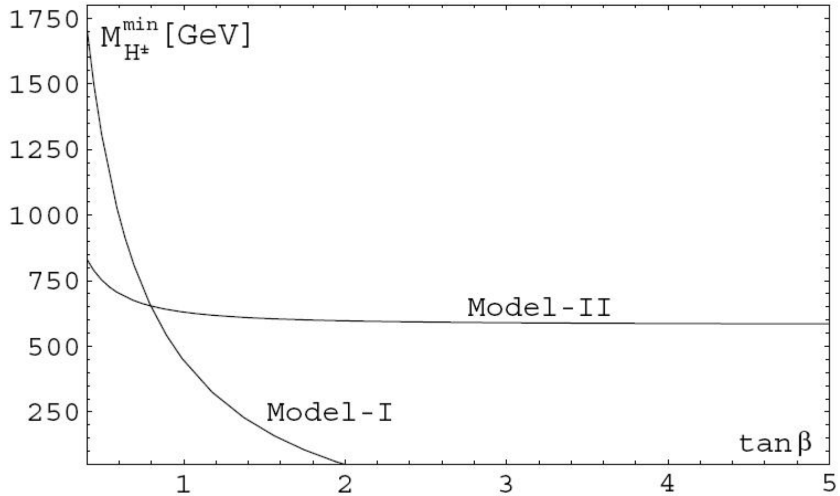
$$- \mathcal{B}_{22}(M_Z^2; M_{H^\pm}^2, M_{H^\pm}^2) - \mathcal{B}_{22}(M_Z^2; M_Z^2, M_{h,\mathrm{ref}}^2) + M_Z^2\,\mathcal{B}_0(M_Z^2; M_Z^2, M_{h,\mathrm{ref}}^2)\Big\},$$

H. E. Haber, D. O'Neil, PRD83 (2011) 055017; A. Celis, V. Ilisie, A. Pich, JHEP07 (2013) 053

$$
\begin{aligned}
U \; &= \mathcal{H}(M_W^2) - \mathcal{H}(M_Z^2) \\
&+ \frac{1}{\pi M_W^2}\Big\{\cos^2(\beta-\alpha)\,\mathcal{B}_{22}(M_W^2; M_{H^\pm}^2, M_h^2) + \sin^2(\beta-\alpha)\,\mathcal{B}_{22}(M_W^2; M_{H^\pm}^2, M_H^2) \\
&+\; \mathcal{B}_{22}(M_W^2; M_{H^\pm}^2, M_A^2) - 2\,\mathcal{B}_{22}(M_W^2; M_{H^\pm}^2, M_{H^\pm}^2)\Big\} \\
&- \frac{1}{\pi M_Z^2}\Big\{\cos^2(\beta-\alpha)\,\mathcal{B}_{22}(M_Z^2; M_h^2, M_A^2) + \sin^2(\beta-\alpha)\,\mathcal{B}_{22}(M_Z^2; M_H^2, M_A^2) \\
&-\; \mathcal{B}_{22}(M_Z^2; M_{H^\pm}^2, M_{H^\pm}^2)\Big\},
\end{aligned}
$$

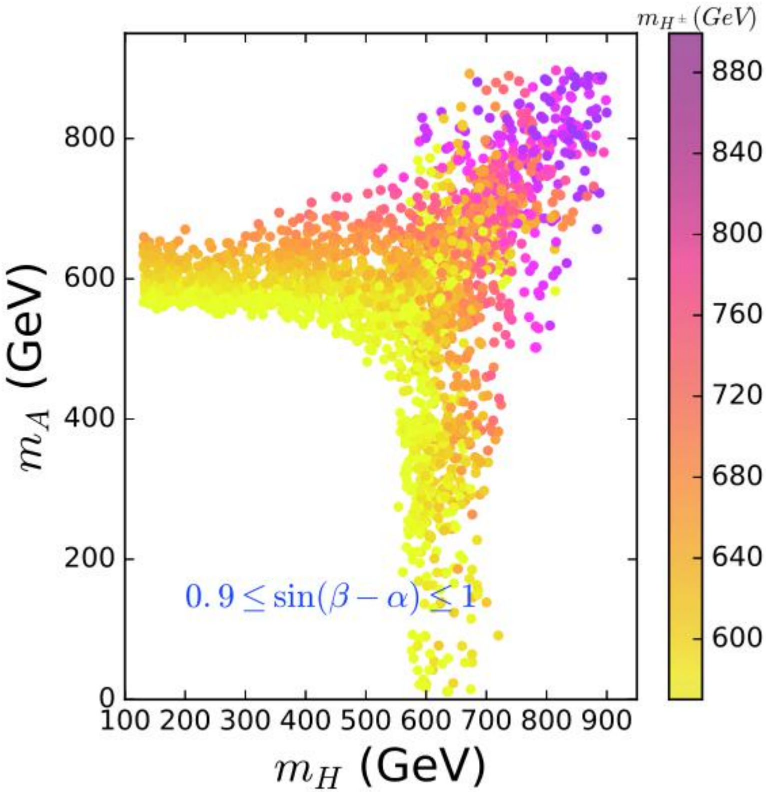
$$
\begin{aligned}
\mathcal{H}(M_V^2) \;\equiv\; \frac{1}{\pi M_V^2}\Big\{&\sin^2(\beta-\alpha)\,\Big[\mathcal{B}_{22}(M_V^2; M_V^2, M_h^2) - M_V^2\,\mathcal{B}_0(M_V^2; M_V^2, M_h^2)\Big] \\
&+\; \cos^2(\beta-\alpha)\,\Big[\mathcal{B}_{22}(M_V^2; M_V^2, M_H^2) - M_V^2\,\mathcal{B}_0(M_V^2; M_V^2, M_H^2)\Big] \\
&-\; \mathcal{B}_{22}(M_V^2; M_V^2, M_{h,\mathrm{ref}}^2) + M_V^2\,\mathcal{B}_0(M_V^2; M_V^2, M_{h,\mathrm{ref}}^2)\Big\}.
\end{aligned}
$$

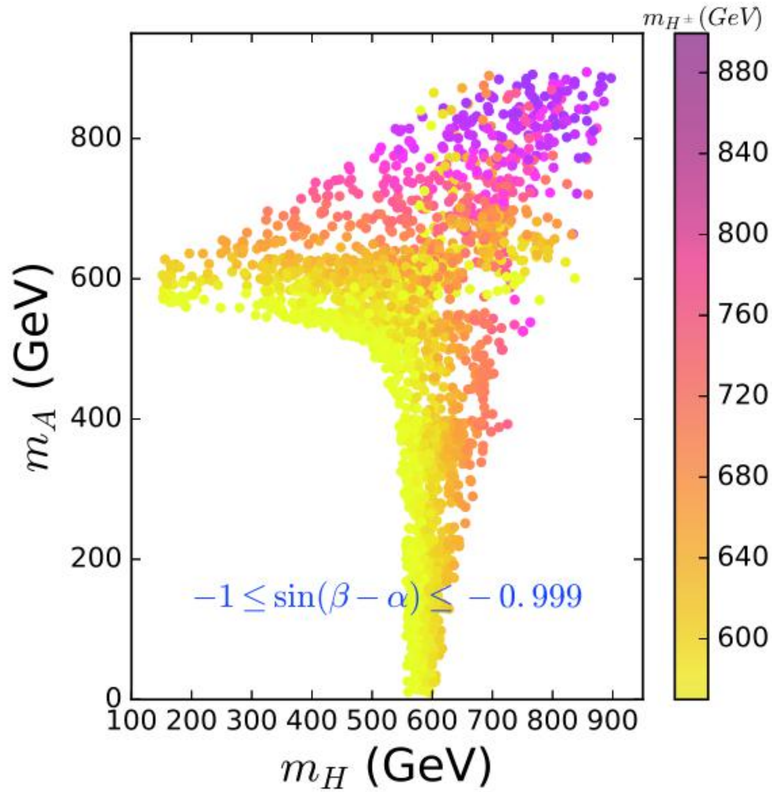
$$
\begin{aligned}
B_{22}(q^2; m_1^2, m_2^2) \;&=\; \frac{1}{4}\,(\Delta+1)\,[m_1^2 + m_2^2 - \frac{1}{3}\,q^2] - \frac{1}{2}\int_0^1 dx\; X\;\log\,(X - i\epsilon)\,, \\
B_0(q^2; m_1^2, m_2^2) \;&=\; \Delta - \int_0^1 dx\;\log\,(X - i\epsilon)\,, \\
\mathcal{F}(m_1^2, m_2^2) \;&=\; \frac{1}{2}\,(m_1^2 + m_2^2) - \frac{m_1^2 m_2^2}{m_1^2 - m_2^2}\;\log\left(\frac{m_1^2}{m_2^2}\right),
\end{aligned}
$$



M. Misiak, M. Steinhauser, EPJC77 (2017) 201

The experimental data of $b \to s\gamma$ requires $m_{H^\pm} > 570$ GeV in the type-II model.





Searches for additional Higgs at the LHC

$$y_V^h = \sin(\beta-\alpha), \quad y_f^h = \sin(\beta-\alpha) + \cos(\beta-\alpha)\kappa_f,$$

$$y_V^H = \cos(\beta-\alpha), \quad y_f^H = \cos(\beta-\alpha) - \sin(\beta-\alpha)\kappa_f,$$

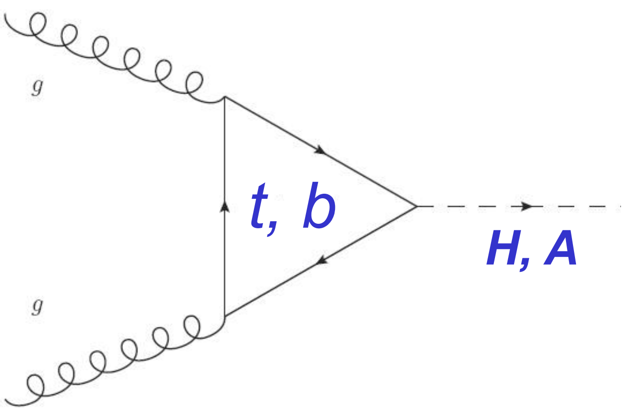
$$y_V^A = 0, \quad y_u^A = -i\gamma^5\kappa_u, \quad y_{d,\ell}^A = i\gamma^5\kappa_{d,\ell},$$

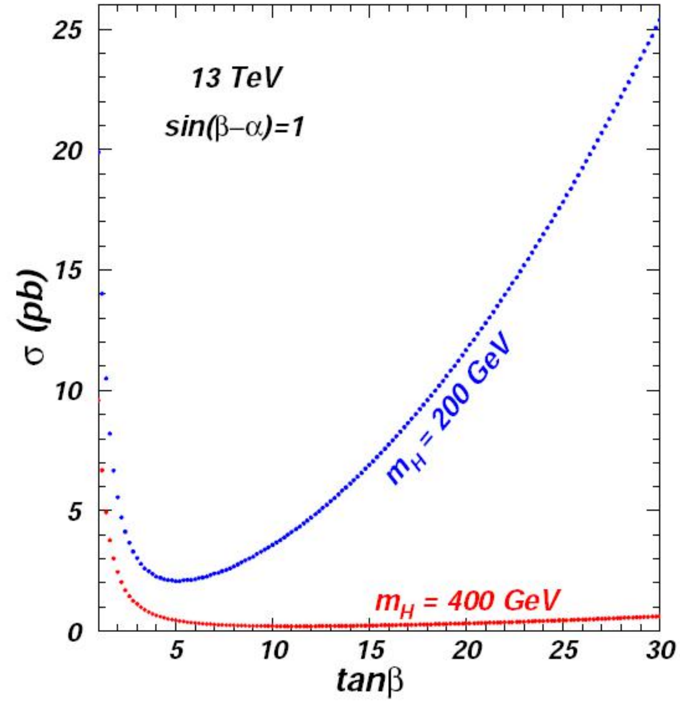
$$\kappa_u = 1/\tan\beta, \quad \kappa_\ell = \kappa_d = -\tan\beta,$$

$$AhZ: -\frac{e}{2s_W c_W}\cos(\beta-\alpha)(p_1-p_2)^\mu$$

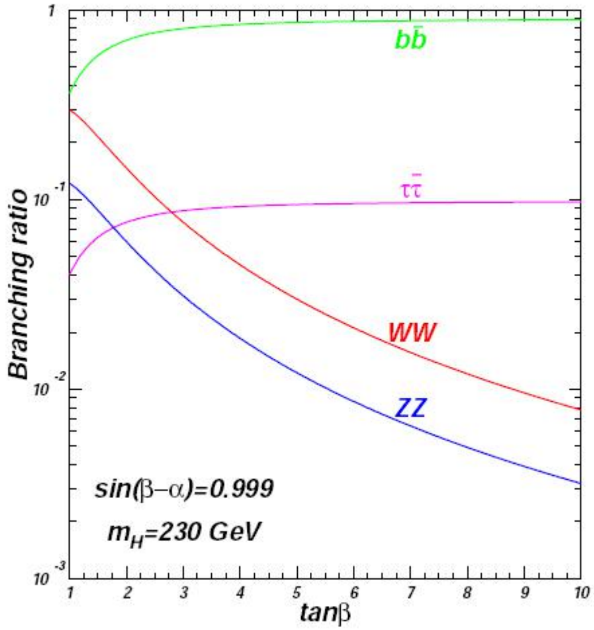
$$AHZ: \frac{e}{2s_W c_W}\sin(\beta-\alpha)(p_1-p_2)^\mu$$

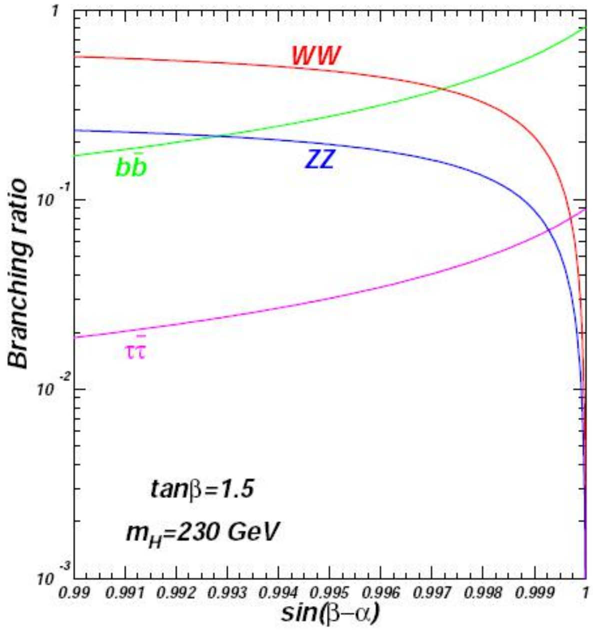
We compute the cross sections for *H* and *A* in the gluon fusion and *bb* fusion production at NNLO in QCD via *SusHi*.

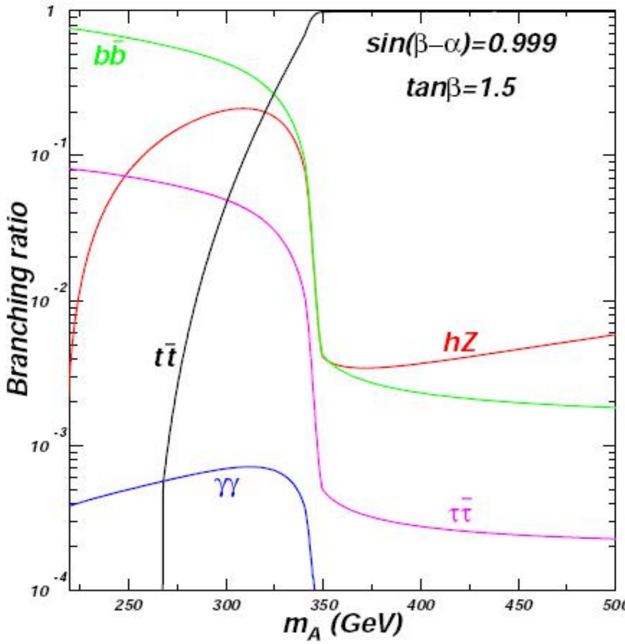




The *2HDMC* is used to calculate the branching ratios of various decay modes of *H* and *A*.

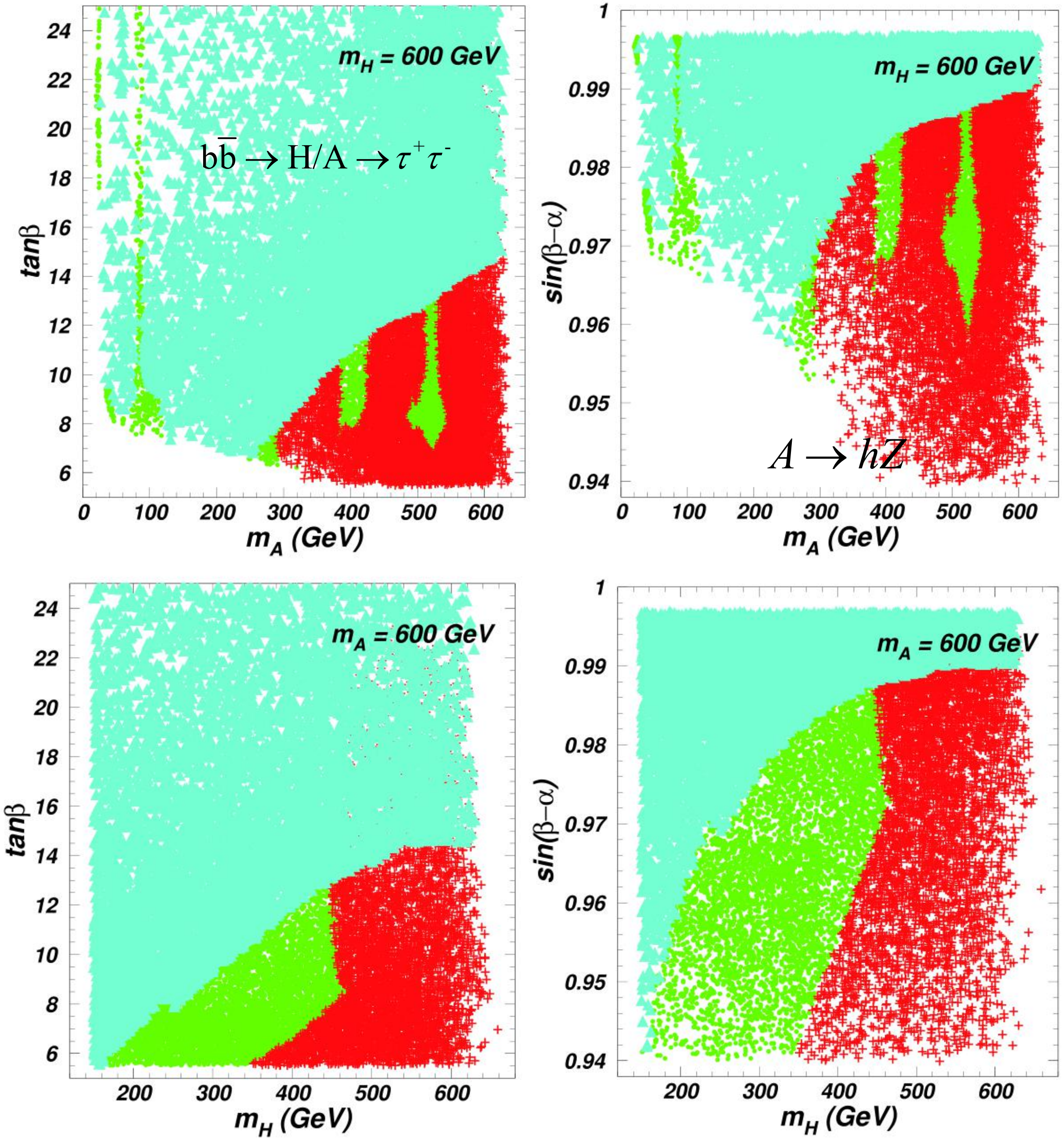




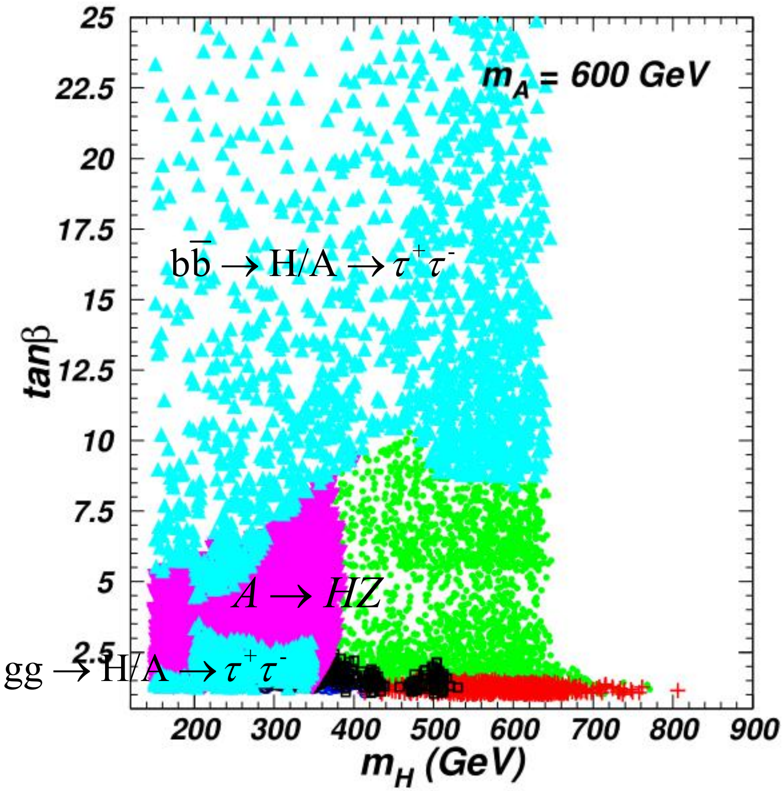


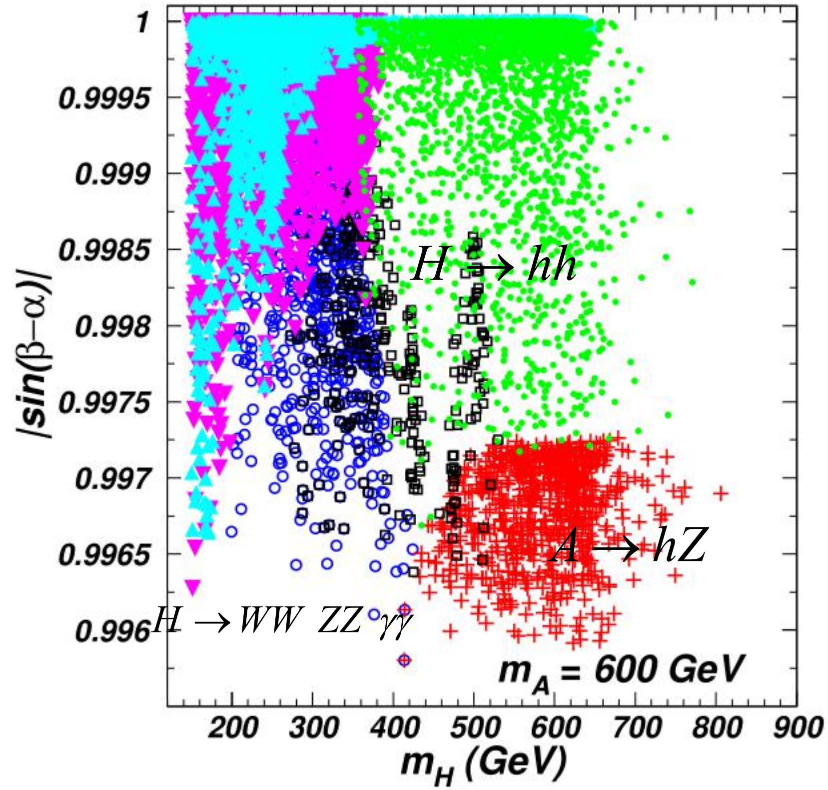
Although the HWW, HZZ and AhZ couplings are suppressed by cos($\beta-\alpha$) , the branching ratios of H $\rightarrow$ WW, H $\rightarrow$ ZZ, and A $\rightarrow$ hZ can be important for a small tanβ.

The case of wrong Sign Yukawa coupling of type-II 2HDM



The case of SM-like Higgs coupling of type-II 2HDM





$$y_V^h = \sin(\beta-\alpha), \quad y_f^h = \sin(\beta-\alpha) + \cos(\beta-\alpha)\kappa_f,$$

$$y_V^H = \cos(\beta-\alpha), \quad y_f^H = \cos(\beta-\alpha) - \sin(\beta-\alpha)\kappa_f,$$

$$y_V^A = 0, \quad y_u^A = -i\gamma^5\kappa_u, \quad y_{d,\ell}^A = i\gamma^5\kappa_{d,\ell},$$

$$\kappa_u = 1/\tan\beta, \quad \kappa_\ell = \kappa_d = -\tan\beta,$$

$$AhZ: -\frac{e}{2s_W c_W}\cos(\beta-\alpha)(p_1-p_2)^\mu$$

$$AHZ: \frac{e}{2s_W c_W}\sin(\beta-\alpha)(p_1-p_2)^\mu$$

Lepton anomalous magnetic moment

muon anomalous magnetic moment:

Result of BNL has 3.7σ positive deviation:

$$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = (274 \pm 73) \times 10^{-11}.$$

Muon g-2 collaboration, G. W. Bennett et al., PRD73 (2006) 072003.

Combining with data of BNL, result of Fermilab has 4.2σ positive deviation:

Muon g-2 collaboration, B. Abi, PRL126 (2021) 141801.

$$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = (251 \pm 59) \times 10^{-11}.$$

electron anomalous magnetic moment:

Result of Berkeley has 2.4σ negative deviation:

$$\Delta a_e = a_e^{exp} - a_e^{SM} = (-87 \pm 36) \times 10^{-14}$$

R. H. Parker, C. Yu, W. Zhong, B. Estey, H. Muller, Science360 (2018) 191

Result of Laboratoire Kastler Brossel is well consistent with SM

L. Morel, Z. Yao, P. Clade, S. Guellati-Khelifa, Nature588 (2020) 61

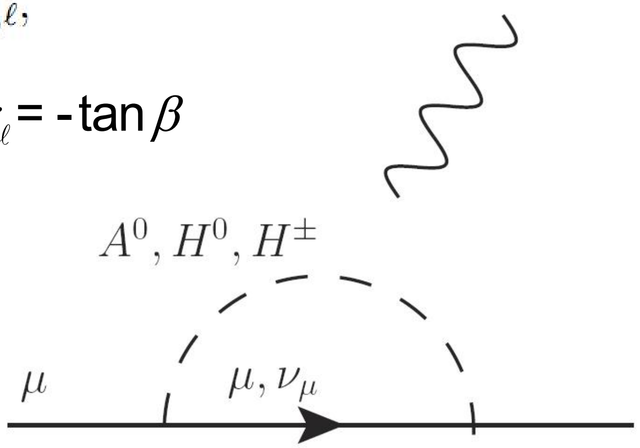
Muon anomalous magnetic moment in lepton-specifc 2HDM (L2HDM)

$$y_V^h = \sin(\beta-\alpha), \quad y_f^h = \sin(\beta-\alpha) + \cos(\beta-\alpha)\kappa_f,$$

$$y_V^H = \cos(\beta-\alpha), \quad y_f^H = \cos(\beta-\alpha) - \sin(\beta-\alpha)\kappa_f,$$

$$y_V^A = 0, \qquad y_u^A = -i\gamma^5\kappa_u, \qquad y_{d,\ell}^A = i\gamma^5\kappa_{d,\ell},$$

Letpon specific 2HDM: $\kappa_u = \kappa_d = 1/\tan\beta, \quad \kappa_\ell = -\tan\beta$



The one loop contribution:

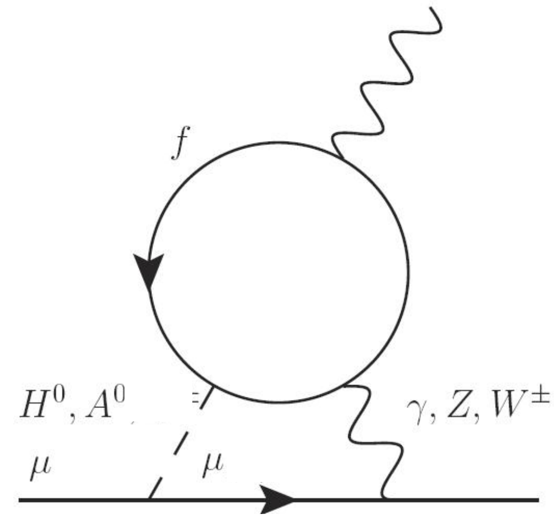
$$\Delta a_\mu^{\rm 2HDM}(1{\rm loop}) = \frac{G_F\, m_\mu^2}{4\pi^2\sqrt{2}} \sum_j \left(y_\mu^j\right)^2 r_\mu^j\, f_j(r_\mu^j),$$

$$f_{h,H}(r) \simeq -\ln r - 7/6, \quad f_A(r) \simeq \ln r + 11/6, \quad f_{H^\pm}(r) \simeq -1/6. \qquad r_\mu^j = m_\mu^2 / m_j^2$$

A. Dedes, H. E. Haber, JHEP0105, (2001) 006.

The contributions of *A*-loop and *H*-loop are negative and positive, respectively.

For the two-loop diagrams, the contributions of diagram mediated by *A* and *H* are positive and negative.



$$\Delta a_\mu^{\rm 2HDM}(2{\rm loop}-{\rm BZ}) = \frac{G_F\, m_\mu^2}{4\pi^2\sqrt{2}}\,\frac{\alpha_{\rm em}}{\pi}\sum_{i,f} N_f^c\, Q_f^2\, y_\mu^i\, y_f^i\, r_f^i\, g_i(r_f^i),$$

$$G_H(r) = \int_0^1 dx\,\frac{2x(1-x)-1}{x(1-x)-r}\ln\frac{x(1-x)}{r},$$

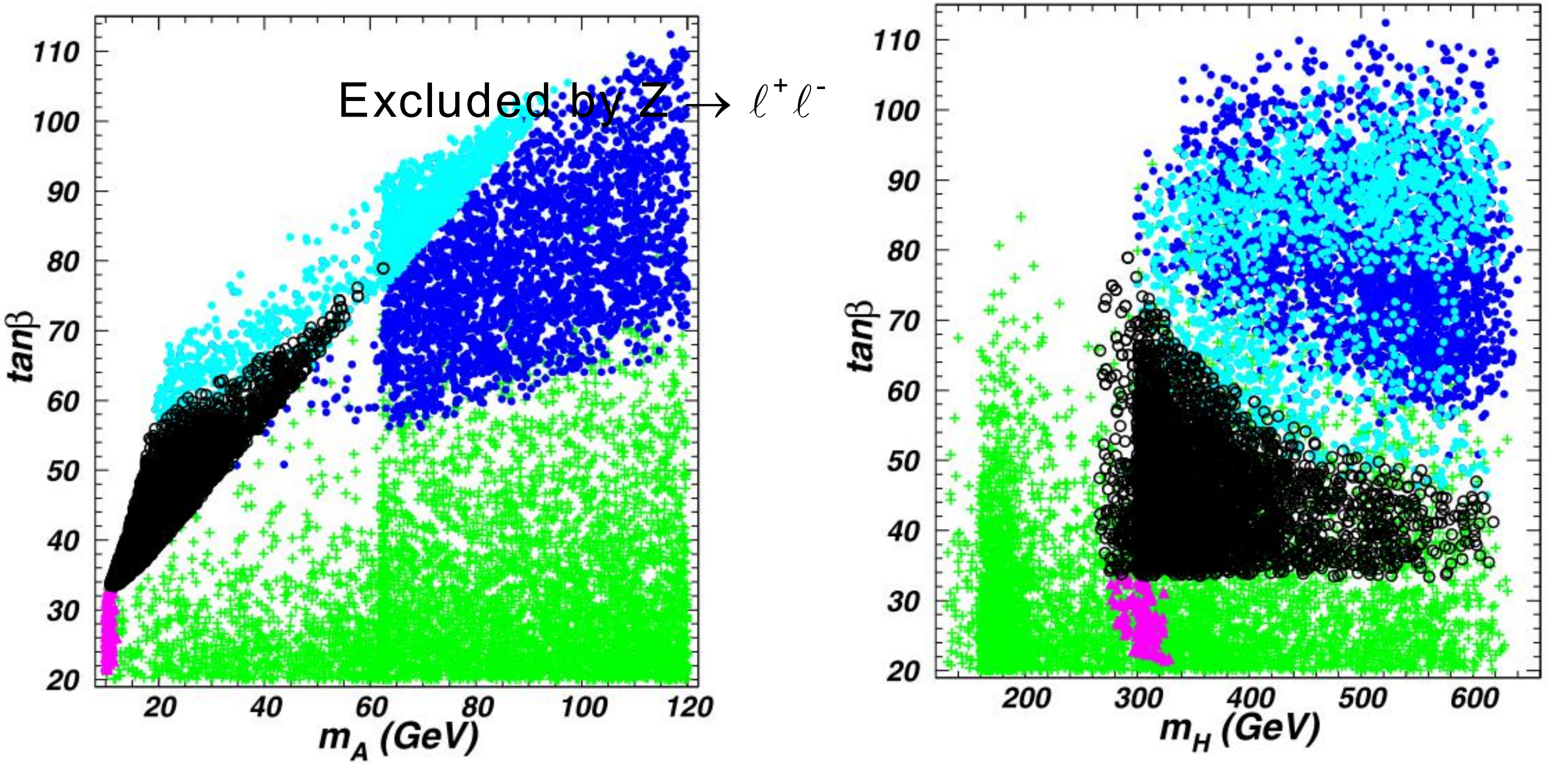
$$G_A(r) = \int_0^1 dx\,\frac{1}{x(1-x)-r}\ln\frac{x(1-x)}{r}.$$

D. Chang, W. F. Chang, C.-H. Chou, W. Y. Keung, PRD63, (2001) 091301.

As m_f^2/m_μ^2 can overcome the loop suppression factor α/π, the two loop contributions may be larger than one-loop ones.

The muon g-2 favors:

- m_A is much smaller than m_H
- A large t_β



L W, j. M. Yang, M. Zhang, Y. Zhang, PLB788 (2019) 519

Constraints from lepton universality in *Z* decays

$$\frac{\Gamma_{Z\to\mu^+\mu^-}}{\Gamma_{Z\to e^+e^-}} = 1.0009 \pm 0.0028\,,$$
$$\frac{\Gamma_{Z\to\tau^+\tau^-}}{\Gamma_{Z\to e^+e^-}} = 1.0019 \pm 0.0032\,,$$

$$\frac{\Gamma_{Z\to\mu^+\mu^-}}{\Gamma_{Z\to e^+e^-}} \approx 1.0 + \frac{2g_L^e \mathrm{Re}(\delta g_L^{\mathrm{2HDM}}) + 2g_R^e \mathrm{Re}(\delta g_R^{\mathrm{2HDM}})}{{g_L^e}^2 + {g_R^e}^2}\frac{m_\mu^2}{m_\tau^2}\,.,$$
$$\frac{\Gamma_{Z\to\tau^+\tau^-}}{\Gamma_{Z\to e^+e^-}} \approx 1.0 + \frac{2g_L^e \mathrm{Re}(\delta g_L^{\mathrm{2HDM}}) + 2g_R^e \mathrm{Re}(\delta g_R^{\mathrm{2HDM}})}{{g_L^e}^2 + {g_R^e}^2}\,.$$

$$\delta g_L^{\mathrm{2HDM}} = \frac{1}{16\pi^2}\frac{m_\tau^2}{v^2}t_\beta^2\Big\{-\frac{1}{2}B_Z(r_A) - \frac{1}{2}B_Z(r_H) - 2C_Z(r_A, r_H) + s_W^2\left[B_Z(r_A) + B_Z(r_H) + \tilde{C}_Z(r_A) + \tilde{C}_Z(r_H)\right]\Big\},$$

$$\delta g_R^{\mathrm{2HDM}} = \frac{1}{16\pi^2}\frac{m_\tau^2}{v^2}t_\beta^2\Big\{2C_Z(r_A, r_H) - 2C_Z(r_{H^\pm}, r_{H^\pm}) + \tilde{C}_Z(r_{H^\pm}) - \frac{1}{2}\tilde{C}_Z(r_A) - \frac{1}{2}\tilde{C}_Z(r_H) + s_W^2\left[B_Z(r_A) + B_Z(r_H) + 2B_Z(r_{H^\pm}) + \tilde{C}_Z(r_A) + \tilde{C}_Z(r_H) + 4C_Z(r_{H^\pm}, r_{H^\pm})\right]\Big\},$$

The extra Higgs bosons are dominantly produced at the LHC via the following electroweak processes,

$$pp \to W^{\pm *} \to H^{\pm} A, \qquad pp \to W^{\pm *} \to H^{\pm} H,$$

$$pp \to Z^*/\gamma^* \to H A, \qquad pp \to Z^*/\gamma^* \to H^+ H^-$$

The main decay modes,

$$A \to \tau^+\tau^-,\ \mu^+\mu^-, \cdots\cdots ,$$

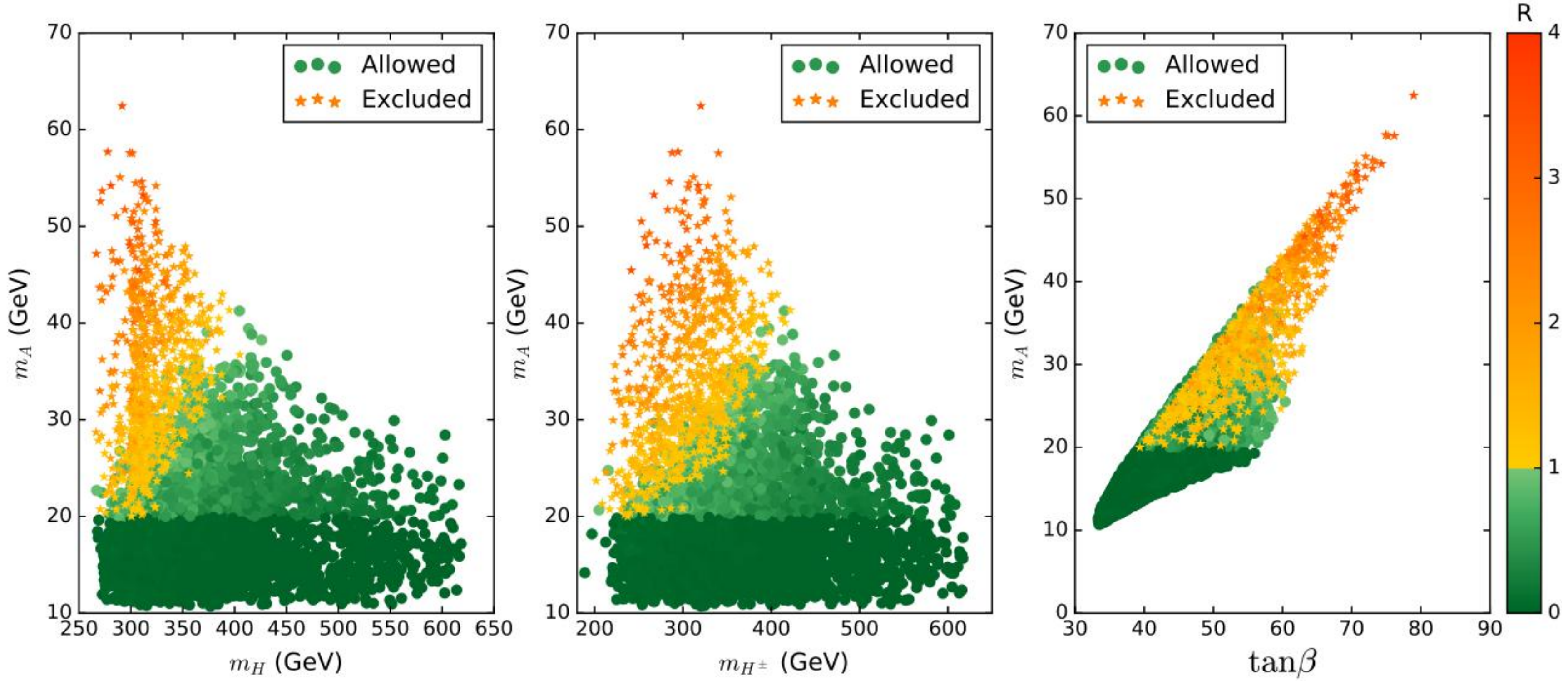
$$H \to \tau^+\tau^-,\ ZA, \cdots\cdots ,$$

$$H^{\pm} \to \tau^{\pm}\nu,\ W^{\pm}A, \cdots\cdots .$$

The dominated final states generated at LHC of our samples ,

$$pp \to W^{\pm *} \to H^{\pm} A \to, 3\tau + \upsilon_{\tau} \text{ or } 4\tau + W^{\pm}$$

$$pp \to Z^*/\gamma^* \to H A \to 4\tau \text{ or } 4\tau + Z.$$

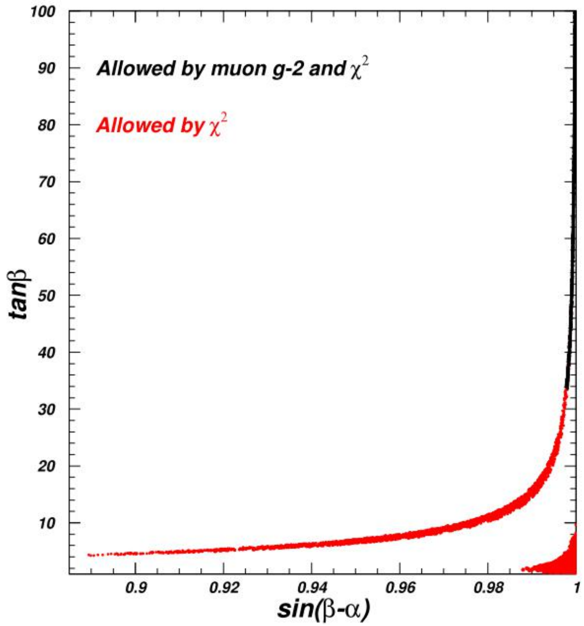


The direct multi-lepton events searches at the LHC reduce the allowed parameter space sizably.

Muon g-2 favors the 125 GeV Higgs with wrong sign Yukawa coupling to lepton in the lepton specific 2HDM

$$y_V^h = \sin(\beta-\alpha) \quad y_f^h = [\sin(\beta-\alpha) + \cos(\beta-\alpha)\kappa_f],$$

$\kappa_u = \kappa_d = 1/\tan\beta, \quad \kappa_\ell = -\tan\beta,$



Vacuum stability requires

$$\lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 > -\sqrt{\lambda_1\lambda_2}, \quad \lambda_3 + \lambda_4 - |\lambda_5| > -\sqrt{\lambda_1\lambda_2}.$$

For $\cos(\beta-\alpha)=0$

$$v^2\lambda_1 = m_h^2 - \frac{t_\beta^3(m_{12}^2 - m_H^2 s_\beta c_\beta)}{s_\beta^2},$$

$$v^2\lambda_2 = m_h^2 - \frac{(m_{12}^2 - m_H^2 s_\beta c_\beta)}{t_\beta s_\beta^2},$$

$$v^2\lambda_3 = m_h^2 + 2m_{H^\pm}^2 - 2m_H^2 - \frac{t_\beta(m_{12}^2 - m_H^2 s}{s_\beta^2}$$

$$v^2\lambda_4 = m_A^2 - 2m_{H^\pm}^2 + m_H^2 + \frac{t_\beta(m_{12}^2 - m_H^2 s_\beta c_\beta)}{s_\beta^2},$$

$$v^2\lambda_5 = m_H^2 - m_A^2 + \frac{t_\beta(m_{12}^2 - m_H^2 s_\beta c_\beta)}{s_\beta^2},$$

For a large t_β, the first condition favors $\left(m_{12}^2 - m_H^2 s_\beta c_\beta\right) \to 0$

Thus, the last condition requires

$$m_h^2 + m_A^2 - m_H^2 > 0.$$

The lepton universality of tau decays:

$$\left(\frac{g_\tau}{g_e}\right)^2 \equiv \bar{\Gamma}(\tau \to \mu\nu\bar{\nu})/\bar{\Gamma}(\mu \to e\nu\bar{\nu}) \qquad \left(\frac{g_\tau}{g_e}\right) = 1.0029 \pm 0.0015$$

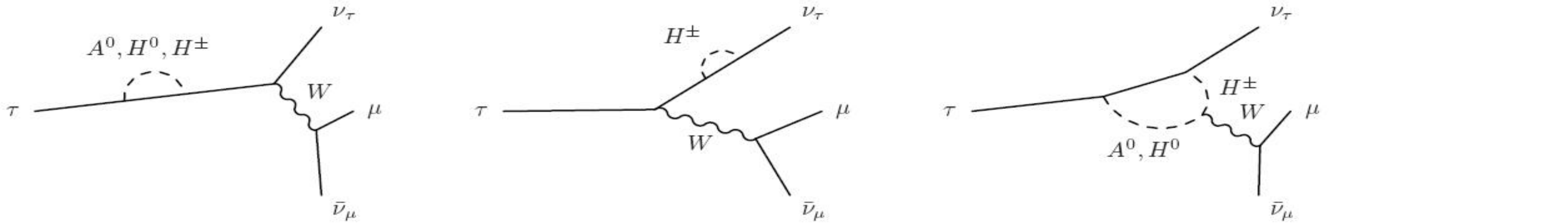
The correction of L2HDM: $\left(\frac{g_\tau}{g_e}\right) \approx 1 + \delta_{\rm tree} + \delta_{\rm loop}$

The tree-level diagram medaited by the charged Higgs can give negative contribution to τ→μνν

$$\delta_{\rm tree} = \frac{m_\tau^2 m_\mu^2}{8 m_{H^\pm}^4} t_\beta^4 - \frac{m_\mu^2}{m_{H^\pm}^2} t_\beta^2 \frac{g(m_\mu^2/m_\tau^2)}{f(m_\mu^2/m_\tau^2)},$$

$$\delta_{\rm loop} = \frac{1}{16\pi^2}\frac{m_\tau^2}{v^2} t_\beta^2 \left[1 + \frac{1}{4}\left(H(x_A) + s_{\beta-\alpha}^2 H(x_H) + c_{\beta-\alpha}^2 H(x_h)\right)\right] \qquad x_\phi = m_\phi^2/m_{H^\pm}^2$$

K. Tobe, JHEP10 (2016) 114;
T. Abe, R. Sato, K. Yagyu, JHEP07 (2015) 064



The sign of $\delta_{\rm tree}$ is negative, and L2HDM raises the discrepancy of g_τ/g_e

Lepton anomalous magnetic moment and lepton-specifc inert 2HDM

We introduce an inert Higgs doublet Φ_2 in the SM as well as a discrete symmetry under which Φ_2 is odd while all the SM particles are even.

$$
\begin{aligned}
\mathrm{V} \;=\; & Y_1(\Phi_1^\dagger\Phi_1) + Y_2(\Phi_2^\dagger\Phi_2) + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 \\
& + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\
& + \left[\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 + \mathrm{h.c.}\right] \; .
\end{aligned}
$$

$$
\Phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\,(v + h + iG_0) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}\,(H + iA) \end{pmatrix}
$$

***Y_1* is determined by requiring the scalar potential minimization condition,**

$$
Y_1 = -\frac{1}{2}\lambda_1 v^2 .
$$

The masses of physical states: *h*, *H*, *A*, $H^\pm$

$$
m_{H^\pm}^2 = Y_2 + \frac{\lambda_3}{2}v^2, \quad m_A^2 = m_{H^\pm}^2 + \frac{1}{2}(\lambda_4 - \lambda_5)v^2
$$

$$
m_h^2 = \lambda_1 v^2 \equiv (125\ \mathrm{GeV})^2, \quad m_H^2 = m_A^2 + \lambda_5 v^2
$$

The fermions obtain the mass terms from the Yukawa interactions with Φ_1,

$$-\mathcal{L} = y_u \overline{Q}_L \, \tilde{\Phi}_1 \, u_R + y_d \overline{Q}_L \, \Phi_1 \, d_R + y_l \overline{L}_L \, \Phi_1 \, e_R + \mathrm{h.c.}$$

Only in the lepton sector we introduce the discrete symmetry-breaking Yukawa interactions with Φ_2,

$$\begin{aligned} -\mathcal{L} \; = \; & \sqrt{2}\, \kappa_e \, \overline{L}_{1L} \, \Phi_2 \, e_R + \sqrt{2}\, \kappa_\mu \, \overline{L}_{2L} \, \Phi_2 \, \mu_R \\ & + \sqrt{2}\, \kappa_\tau \, \overline{L}_{3L} \, \Phi_2 \, \tau_R + \mathrm{h.c.}\,. \end{aligned}$$

The inert Higgses (*H, A, H*$^\pm$) only have couplings to the lepton. Their couplings to quarks and gauge bosons are absent. The 125 GeV Higgs has the same couplings as the SM at the tree-level.

Muon g-2:

$$\Delta a_\mu^{\rm 2HDM}(1{\rm loop}) = \frac{m_\mu^2}{8\pi^2 v^2} \sum_i \kappa_\mu^2 \, r_\mu^i \, F_j(r_\mu^i),$$

$$i = h,\ A,\ H^\pm,\ r_\mu^i = m_\mu^2/M_j^2.$$

$$f_{h,H}(r) \simeq -\ln r - 7/6, \quad f_A(r) \simeq \ln r + 11/6, \quad f_{H^\pm}(r) \simeq -1/6.$$

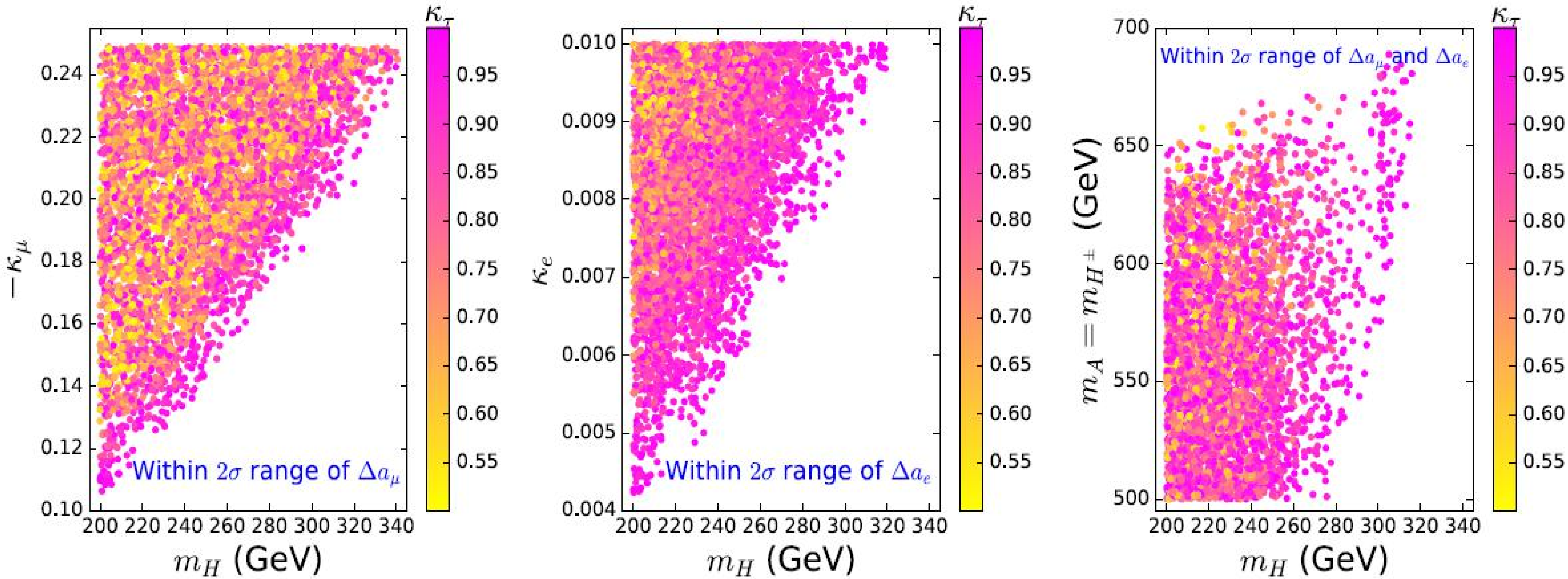
$$\Delta a_\mu^{\rm 2HDM}(2{\rm loop}) = \frac{m_\mu^2}{8\pi^2 v^2} \frac{\alpha_{\rm em}}{\pi} \sum_{i,\ell} Q_\ell^2 \, \kappa_\mu \, \kappa_\ell \, r_\ell^i \, G_i(r_\ell^i),$$

$$G_h(r) = \int_0^1 dx \, \frac{2x(1-x)-1}{x(1-x)-r} \ln \frac{x(1-x)}{r},$$

$$G_A(r) = \int_0^1 dx \, \frac{1}{x(1-x)-r} \ln \frac{x(1-x)}{r}.$$

The calculation of electron g-2 is similar to muon g-2.

For $m_H < m_A$ and $k_\mu k_\tau < 0$, both one-loop and two-loop diagrams give positive contributions to muon g-2. For $m_H < m_A$ and $k_e k_\tau > 0$, one-loop and two-loop diagrams give positive and negative contributbgins to electron g-2.


$-\kappa_\mu$
m_H (GeV)
Within 2σ range of Δa_μ
κ_τ
κ_e
Within 2σ range of Δa_e
$m_A = m_{H^\pm}$ (GeV)
Within 2σ range of Δa_μ and Δa_e

Fit to lepton universality of tau decays

$$\left(\frac{g_\tau}{g_\mu}\right) = 1.0011 \pm 0.0015, \quad \left(\frac{g_\tau}{g_e}\right) = 1.0029 \pm 0.0015,$$

$$\left(\frac{g_\mu}{g_e}\right) = 1.0018 \pm 0.0014, \left(\frac{g_\tau}{g_\mu}\right)_\pi = 0.9963 \pm 0.0027,$$

$$\left(\frac{g_\tau}{g_\mu}\right)_K = 0.9858 \pm 0.0071,$$

$$\left(\frac{g_\tau}{g_\mu}\right)^2 \equiv \bar{\Gamma}(\tau \to e\nu\bar{\nu})/\bar{\Gamma}(\mu \to e\nu\bar{\nu}) \approx \frac{1 + 2\delta^\tau_{\rm loop}}{1 + 2\delta^\mu_{\rm loop}},$$

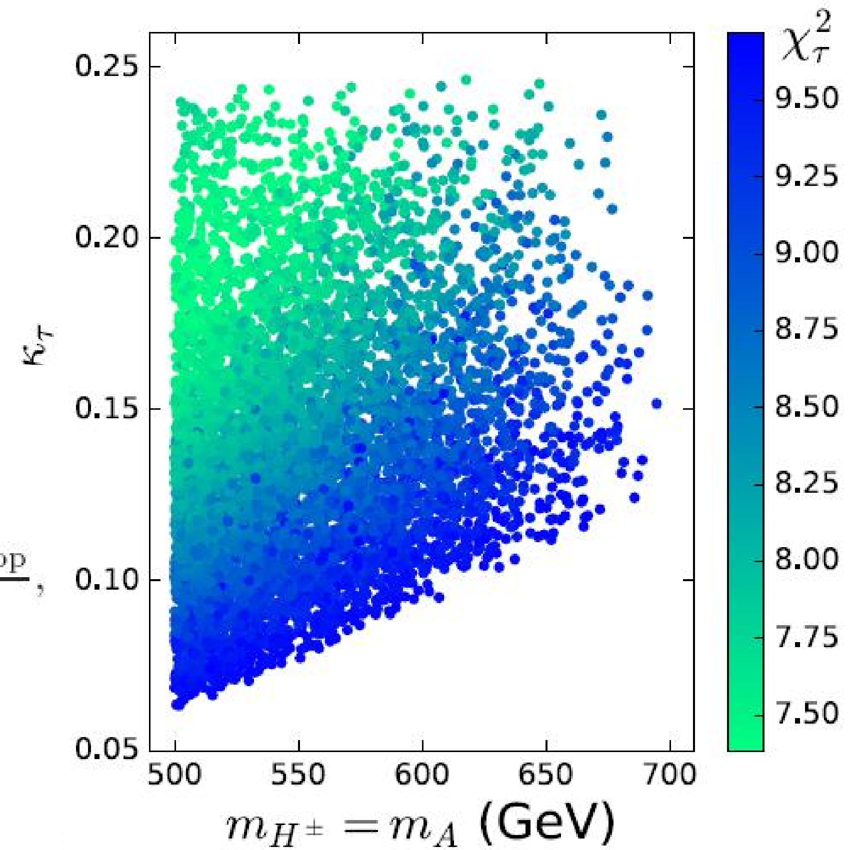
$$\left(\frac{g_\tau}{g_e}\right)^2 \equiv \bar{\Gamma}(\tau \to \mu\nu\bar{\nu})/\bar{\Gamma}(\mu \to e\nu\bar{\nu}) \approx \frac{1 + 2\delta_{\rm tree} + 2\delta^\tau_{\rm loop}}{1 + 2\delta^\mu_{\rm loop}},$$

$$\left(\frac{g_\mu}{g_e}\right)^2 \equiv \bar{\Gamma}(\tau \to \mu\nu\bar{\nu})/\bar{\Gamma}(\tau \to e\nu\bar{\nu}) \approx 1 + 2\delta_{\rm tree},$$

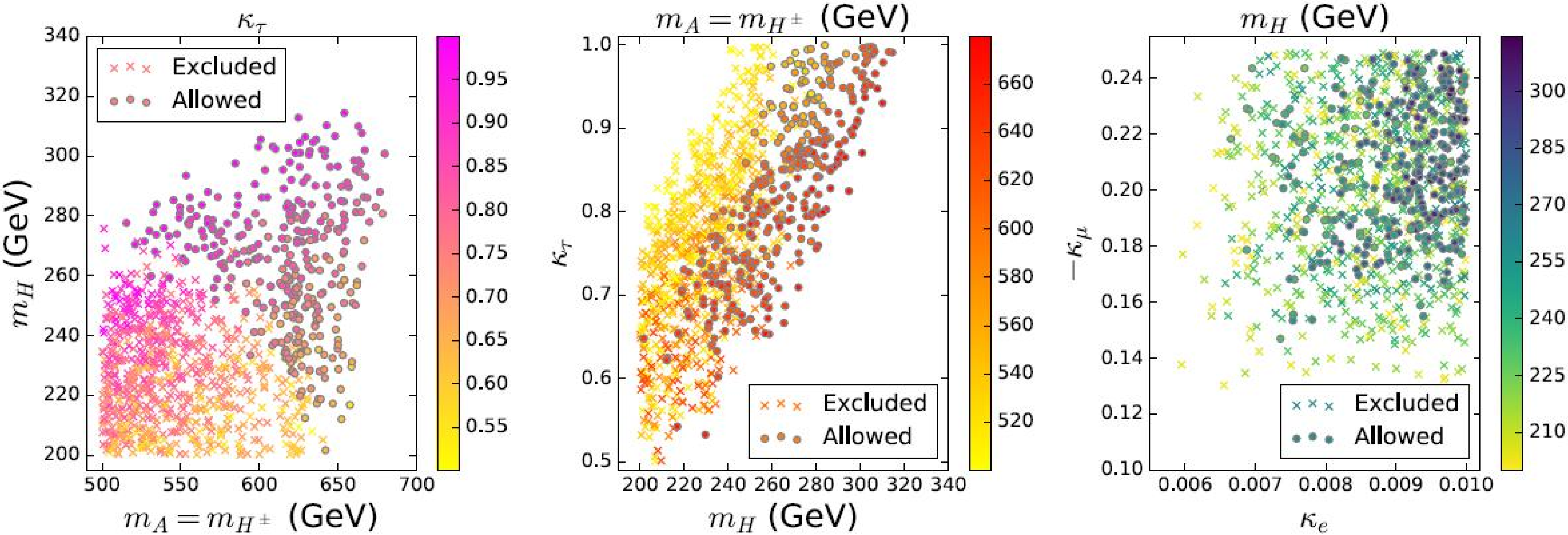
$$\left(\frac{g_\tau}{g_\mu}\right)^2_\pi = \left(\frac{g_\tau}{g_\mu}\right)^2_K = \left(\frac{g_\tau}{g_\mu}\right)^2.$$

$$\delta_{\rm tree} = \frac{v^4 \kappa_\tau^2 \kappa_\mu^2}{8 m_{H^\pm}^4} - \frac{v^2 m_\mu}{m_{H^\pm}^2 m_\tau} \kappa_\tau \kappa_\mu \frac{g(m_\mu^2/m_\tau^2)}{f(m_\mu^2/m_\tau^2)},$$

$$\delta^{\tau,\mu}_{\rm loop} = \frac{1}{16\pi^2} \kappa_{\tau,\mu}^2 \left[1 + \frac{1}{4}\left(H(x_A) + H(x_H)\right)\right]$$

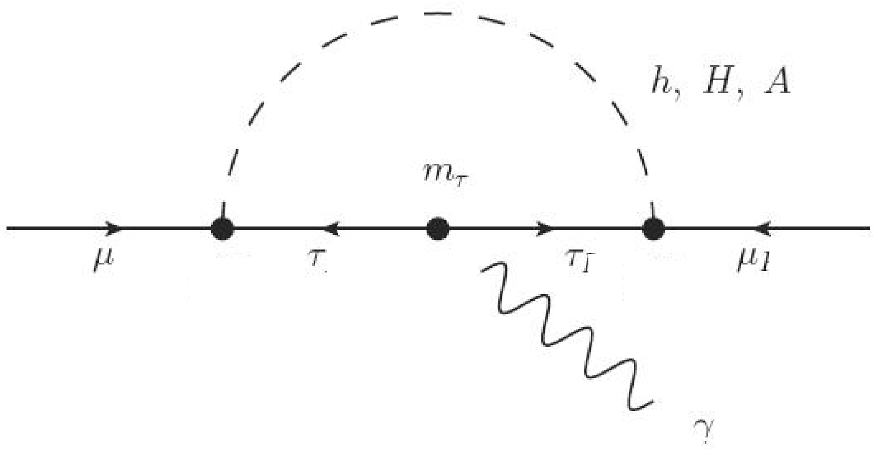


The tree-level diagram medaited by the charged Higgs can give positive contribution to τ→μνν for $k_\mu < 0$.



The allowed and excluded samples by the direct search limits from LHC at 95% confidence level. All the samples simultaneously explain the anomalies of muon g-2, electron g-2, and the data of lepton universality in tau decays, while the constraints of the theory, the oblique parameters, and Z leptonic decays are satisfied.

Muon g-2 and muon-tau-philic Higgs coupling



In the Higgs basis, one can obtain the relevant couplings easily

$$-\mathcal{L} = \sqrt{2}\frac{m_\ell}{v}\,\overline{L}_L\,H_1\,e_R + Y_\ell\,\overline{L}_L\,H_2\,e_R + \text{h.c.}\,,$$

$$H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v + h + iG\right) \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}\left(H + iA\right) \end{pmatrix}$$

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos(\beta-\alpha) & \sin(\beta-\alpha) \\ -\sin(\beta-\alpha) & \cos(\beta-\alpha) \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}$$

We can obtain the following couplings,

$$
\begin{aligned}
-\mathcal{L}_Y = & \left[\frac{m_{\ell_i}}{v}\sin(\beta-\alpha)\delta_{ij} + \cos(\beta-\alpha)\frac{Y_\ell^{ij}}{\sqrt{2}}\right] \; h\bar{\ell}_i\ell_j \\
& + \left[\frac{m_{\ell_i}}{v}\cos(\beta-\alpha)\delta_{ij} - \sin(\beta-\alpha)\frac{Y_\ell^{ij}}{\sqrt{2}}\right] \; H\bar{\ell}_i\ell_j \\
& + i\frac{Y_\ell^{ij}}{\sqrt{2}} \; A\bar{f}_i\gamma_5 f_j + Y_\ell^{ij} H^+ \; \bar{\nu}_i P_R \ell_j + h.c.
\end{aligned}
$$

where $Y_\ell^{\mu\tau} = Y_\ell^{\tau\mu} = \sqrt{2}\rho$

Signal data of 125 GeV Higgs require,

$$| \sin(\beta-\alpha) | \to 1 \text{ for } m_h = 125 \text{ GeV}$$

The muon-tau-philic Higgs interaction can be naturally obtained in the 2HDM with *Z4* discrete symmetry.

Y. Abe, T. Toma, K. Tsumura, JHEP06 (2019) 142

	Q_L^i	U_R^i	D_R^i	L_L^e	L_L^μ	L_L^τ	e_R	μ_R	τ_R	Φ_1	Φ_2
Z_4	1	1	1	1	i	$-i$	1	i	$-i$	1	-1

The scalar potential with a *Z4* discrete symmetry, which is the same as to inert Higgs doublet model.

$$V = Y_1(\Phi_1^\dagger\Phi_1) + Y_2(\Phi_2^\dagger\Phi_2) + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \left[\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right].$$

$$\Phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v+h+iG^0) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H+iA) \end{pmatrix}$$

The Φ_2 field has no VeV.

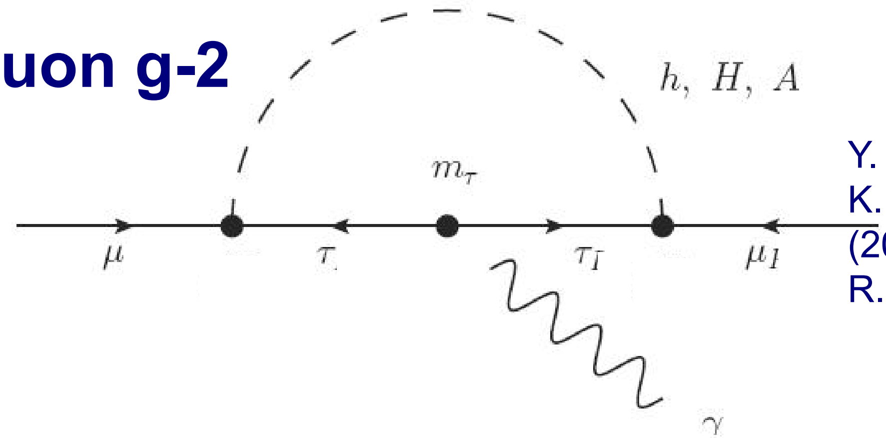
The fermion mass are given via the Yukawa interaction with Φ_1.

$$-\mathcal{L} = y_u\overline{Q}_L\,\tilde{\Phi}_1\,U_R + y_d\overline{Q}_L\,\Phi_1\,D_R + y_\ell\overline{L}_L\,\Phi_1\,E_R + \text{h.c.}$$

The *Z4* symmetry allows Φ_2 to have muon-tau LFV interaction,

$$-\mathcal{L}_{LFV} = \sqrt{2}\,\rho_{\mu\tau}\,\overline{L_L^\mu}\,\Phi_2\,\tau_R + \sqrt{2}\,\rho_{\tau\mu}\,\overline{L_L^\tau}\,\Phi_2\,\mu_R + \text{h.c.}$$

muon g-2



Y. Zhou, Y.-L. Wu, EPJC27 (2003) 577;
K. A. Assamagan, A. Deandrea, P.-A. Delsart, PRD67 (2003) 035001
R. Benbrik, C.-H. Chen, T. Nomura, PRD93 (2016) 9.

$$\delta a_\mu \simeq \frac{m_\mu m_\tau \rho^2}{8\pi^2}\left[\frac{c^2_{\beta\alpha}(\log\frac{m_h^2}{m_\tau^2}-\frac{3}{2})}{m_h^2}+\frac{s^2_{\beta\alpha}(\log\frac{m_H^2}{m_\tau^2}-\frac{3}{2})}{m_H^2}-\frac{\log\frac{m_A^2}{m_\tau^2}-\frac{3}{2}}{m_A^2}\right]$$

$$|\sin(\beta-\alpha)| \to 1 \text{ for } m_h = 125 \text{ GeV}$$

The model may provide a better fit to the lepton universality of tau decays since the tree-level diagram medaited by the charged Higgs can give positive contribution to τ→μνν

$$\bar{\Gamma}(\tau \to \mu\nu\bar{\nu}) = (1+\delta^\tau_{\rm loop})^2\,(1+\delta^\mu_{\rm loop})^2 + \delta_{\rm tree},$$

$$\delta_{\rm tree} = 4\frac{m_W^4 \rho^4}{g^4 m_{H^\pm}^4}. \quad \delta^\tau_{\rm loop} = \delta^\mu_{\rm loop} = \frac{1}{16\pi^2}\rho^2\left[1+\frac{1}{4}\left(H(x_A)+H(x_H)\right)\right]$$

DM phenomenology in inert Higgs doublet model

Physical scalars: inert Higgses H, A, $H^{\pm}$, **and the SM-like Higgs *h***

$$m_{H^\pm}^2 = m_{22}^2 + \frac{\lambda_3}{2} v^2, \quad m_A^2 = m_{H^\pm}^2 + \frac{1}{2}(\lambda_4 - \lambda_5) v^2.$$

$$m_h^2 = \lambda_1 v^2 \equiv (125 \text{ GeV})^2, \quad m_H^2 = m_A^2 + \lambda_5 v^2.$$

H **is the lightest component of inert Higgs and as a DM candiate, which requires**

$$\lambda_5 < 0, \lambda_4 - |\lambda_5| < 0$$

If *A* is a DM candidate, $\lambda_5 > 0$

The parameters which play key roles in DM phenomenology,

$$m_H, \quad m_A, \quad m_{H^\pm}, \quad \lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$$

λ_{345} **controls the *hHH* coupling.**

In addition to the constraints from the vacuum stability, perturbativity, unitarity, oblique parameter, Higgs signal of 125 GeV Higgs, one requires

Gauge boson width :

$$M_{A,H} + M_{H^\pm} \geq M_W,\ M_A + M_H \geq M_Z,\ 2\,M_{H^\pm} \geq M_Z.$$

Null searches from LEP, Tevatron, and LHC:

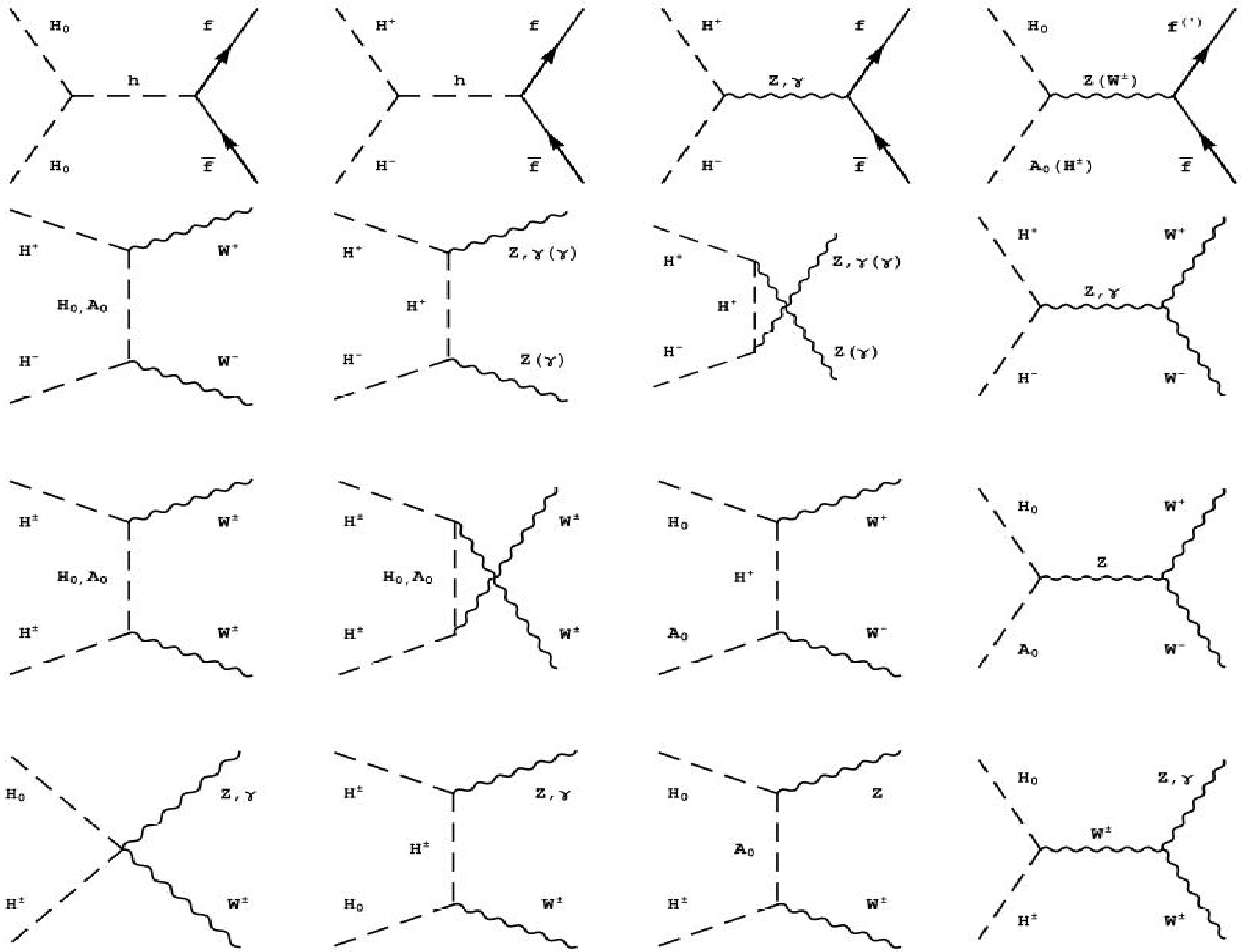
$$M_{H^\pm} \geq 70 \ \text{GeV}.$$

$$M_A \leq 100 \ \text{GeV},\ M_H \leq 80 \ \text{GeV},\ \Delta\, M(A,H) \geq 8 \ \text{GeV},$$

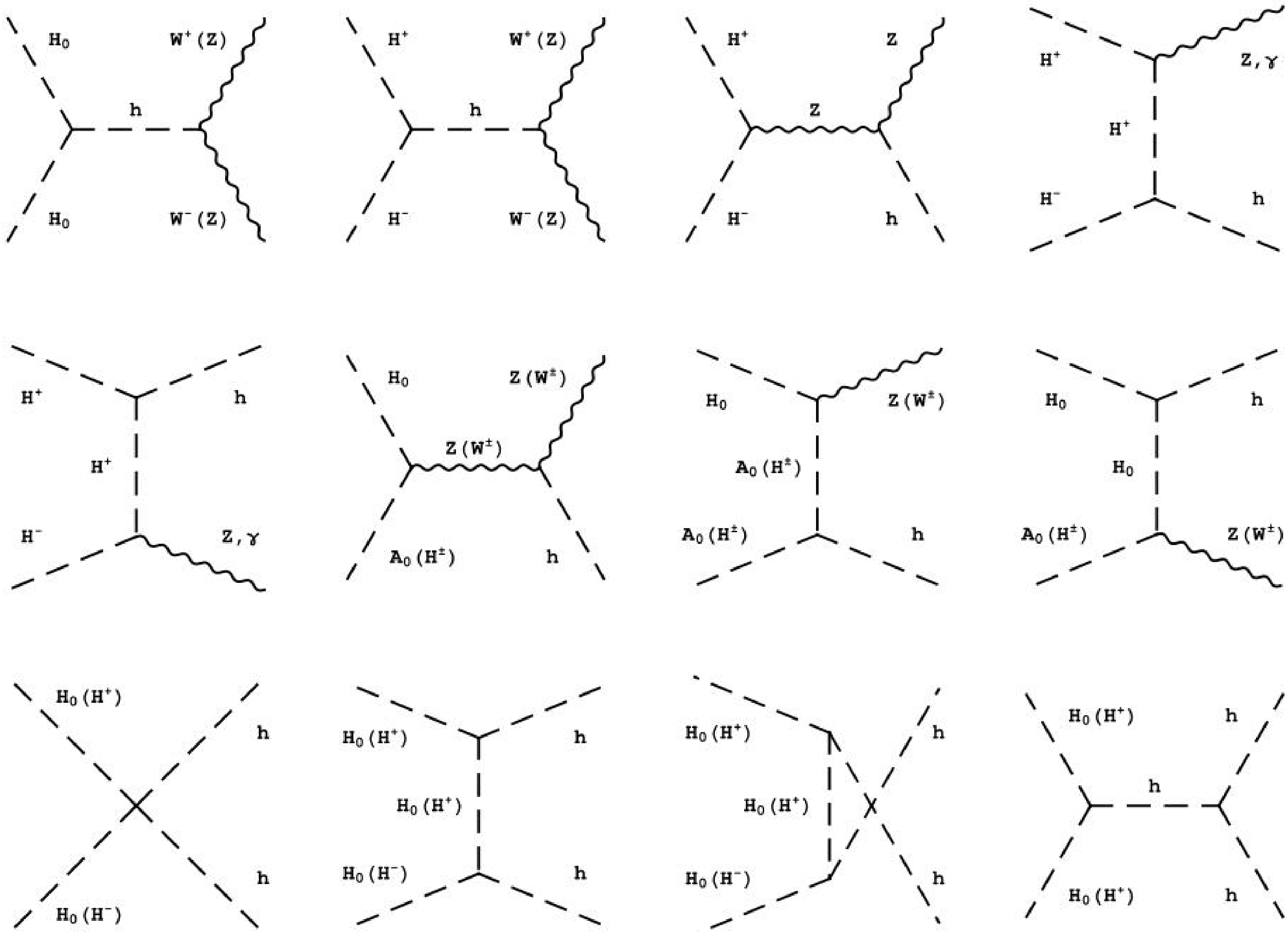
Main annihilation channels:

$$HH \to \mathrm{f}\bar{\mathrm{f}},\ \ HH \to WW^{(*)},\ \ ZZ^{(*)},\ \ HH \to hh$$

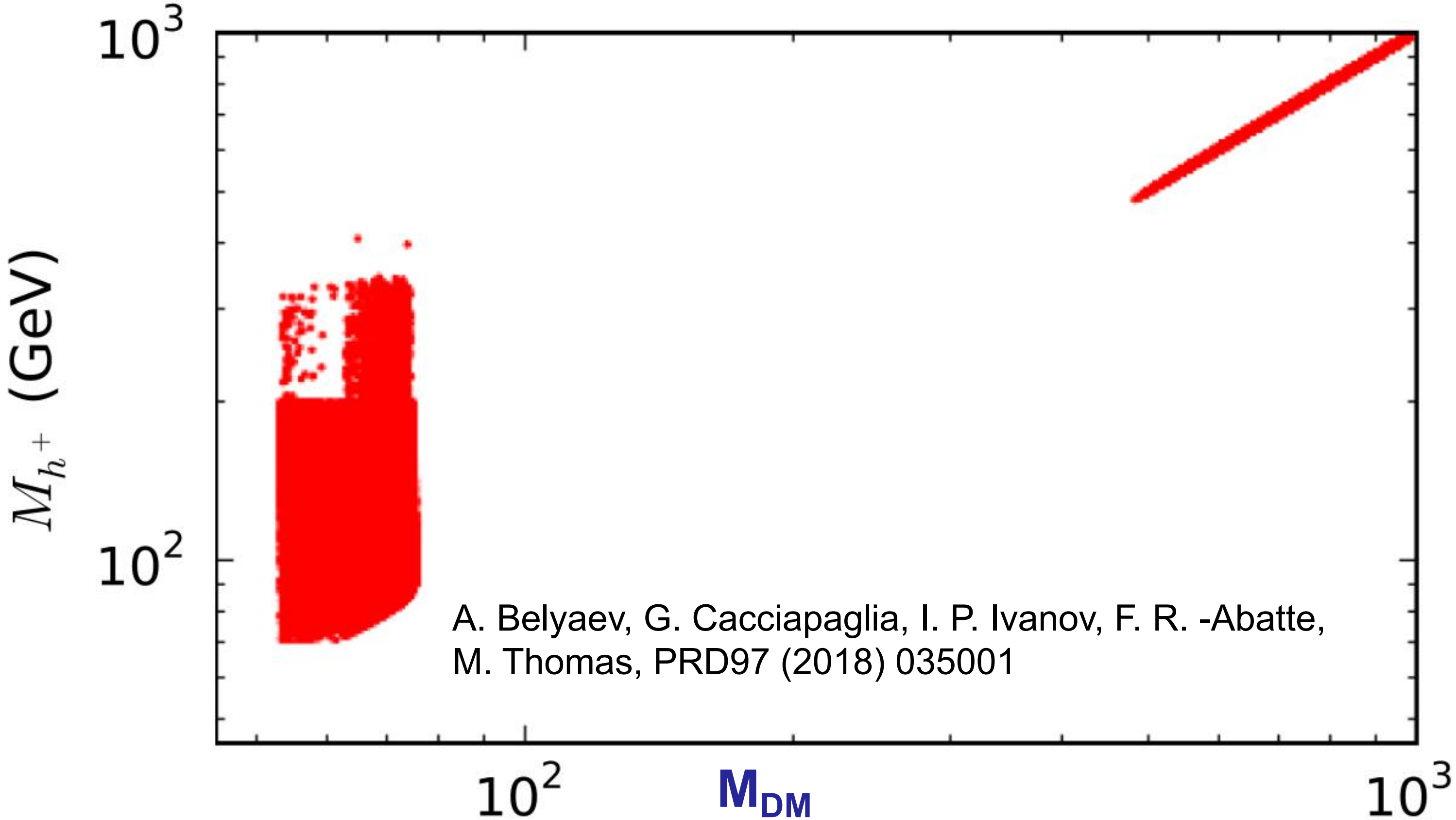
and various relevant co-annihilation of the inert scalars.



only dependent on masses of inert scalar



T. Hambye, F.-S. Ling, L. L. Honorez, J. Rocher, JHEP07 (2009) 090



M_{DM} < 55 GeV: tension between Br(h->HH) and relic density

75 GeV < M_{DM} < 160 GeV: the value of λ_{345} required is ruled out by the limits of direct detection

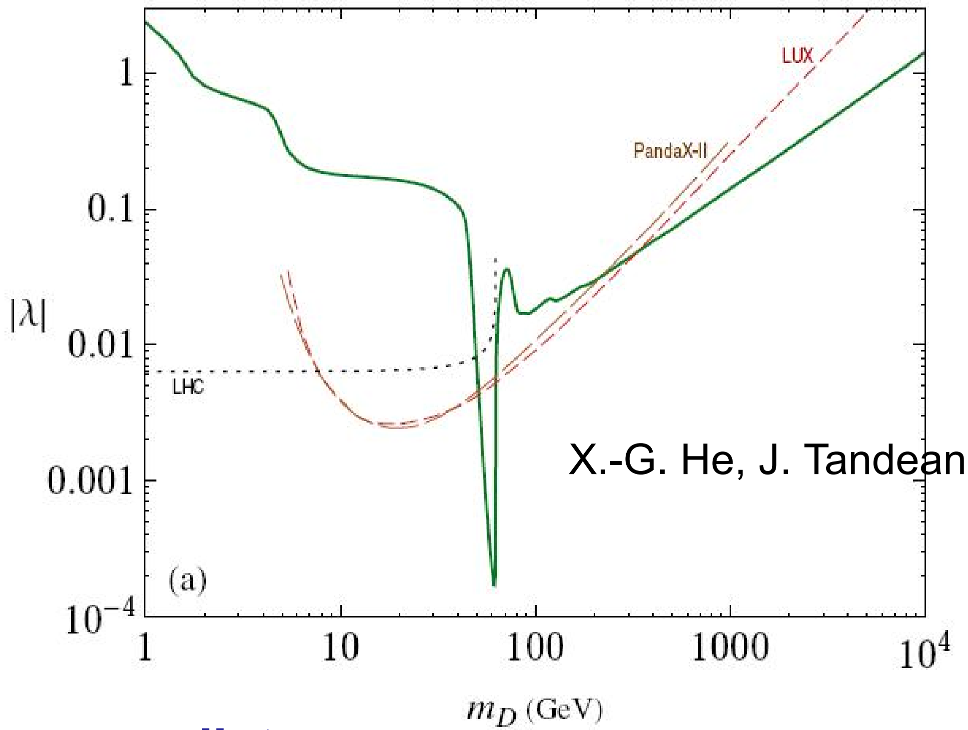
160 GeV < M_{DM} < 500 GeV: the annihilation rates of HH->W^+W^- is too large to rend the exact relic density.

Type-II 2HDM with a singlet scalar DM

- **SM + DM**

  S is real singlet scalar DM, and h is the portal between DM and SM sector V. Silveira and A. Zee, Phys. Lett. B161 (1985) 136

$$\mathcal{L}_S = \frac{1}{2}\partial^\mu S \partial_\mu S - \frac{m_0^2}{2} SS - \frac{\kappa_1}{2}\Phi^\dagger \Phi SS - \frac{\kappa_s}{4!} S^4$$



X.-G. He, J. Tandean, JHEP1612, 074 2016

- **SM + DM + new mediator**

We introduce a real singlet scalar *S* to the type-II 2HDM, and *S* is a possible DM candidate.

$$\mathcal{L}_S = -\frac{1}{2}S^2(\lambda_1\Phi_1^\dagger\Phi_1 + \lambda_2\Phi_2^\dagger\Phi_2) - \frac{m_0^2}{2}S^2 - \frac{\lambda_S}{4!}S^4.$$

$$m_S^2 = m_0^2 + \frac{1}{2}\lambda_1 v^2\cos^2\beta + \frac{1}{2}\lambda_2 v^2\sin^2\beta,$$

$$-\lambda_h v S^2 h/2 \equiv -(-\lambda_1\sin\alpha\cos\beta + \lambda_2\cos\alpha\sin\beta)vS^2h/2,$$

$$-\lambda_H v S^2 H/2 \equiv -(\lambda_1\cos\alpha\cos\beta + \lambda_2\sin\alpha\sin\beta)vS^2H/2.$$

X.-G. He, J. Tandean, PRD88, 013020 (2013); JHEP1612, 074 (2016)
Y. Cai, T. Li, PRD88, 115004 (2013)
A. Drozd, B. Grzadkowski, J. F. Gunion, Y. Jiang, JHEP1411, 105 (2014); JCAP1610, 040 (2016)
L. Wang, R. Shi, X.-F. Han, PRD96 (2017) 115025

The possible DM annihilation channels:

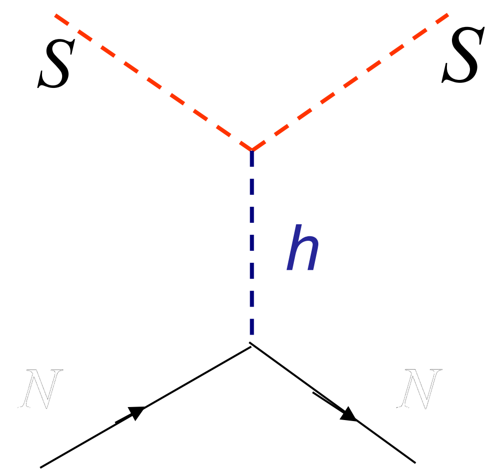
$$SS \to f\bar{f},\ WW^{(*)},\ ZZ^{(*)},\ hh,\ HH,\ AA, H^+H^- \cdots\cdots$$

Isospin-violating DM interactions with nucleons

The 125 GeV Higgs with the wrong sign Yukawa coupling as the portal between the DM and SM sectors

$$\sigma_{p(n)} = \frac{\mu^2_{p(n)}}{4\pi m_S^2}\left[f^{p(n)}\right]^2$$

$$\mu_{p(n)} = \frac{m_S m_{p(n)}}{m_S + m_{p(n)}},$$



$$f^{p(n)} = \sum_{q=u,d,s} f_q^{p(n)} \mathcal{C}_{Sq} \frac{m_{p(n)}}{m_q} + \frac{2}{27} f_g^{p(n)} \sum_{q=c,b,t} \mathcal{C}_{Sq} \frac{m_{p(n)}}{m_q},$$

$$\mathcal{C}_{Sq} = \frac{\lambda_h m_q}{m_h^2} y_q$$

The isospin-violating DM-nucleon coupling can weaken the constraint of DM-nucleon cross section

→ **the natural abundance of the i-th isotope.** (η_i)

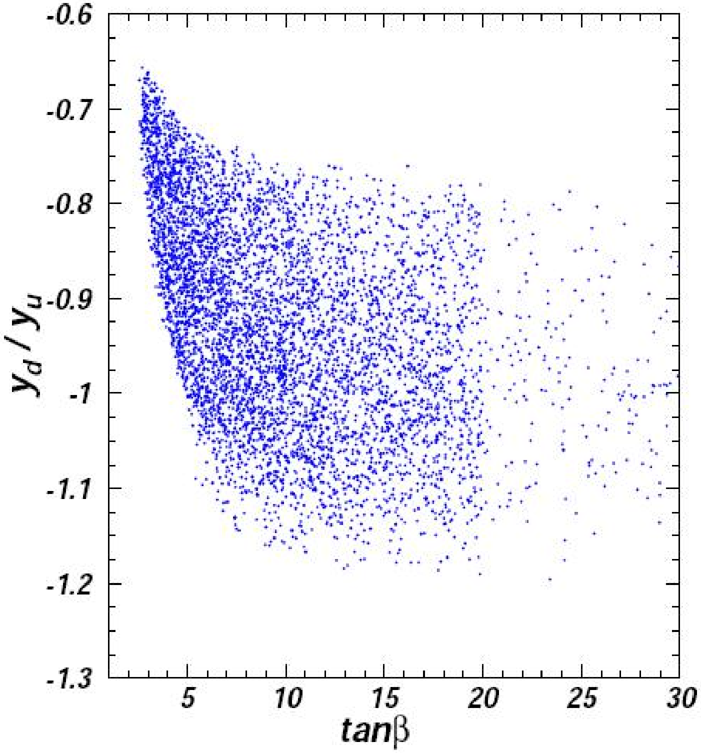
$$\frac{\sigma_p}{\sigma_N^Z} = \frac{\sum_i \eta_i \mu_{A_i}^2 A_i^2}{\sum_i \eta_i \mu_{A_i}^2 [Z + (A_i - Z) f_n / f_p]^2},$$

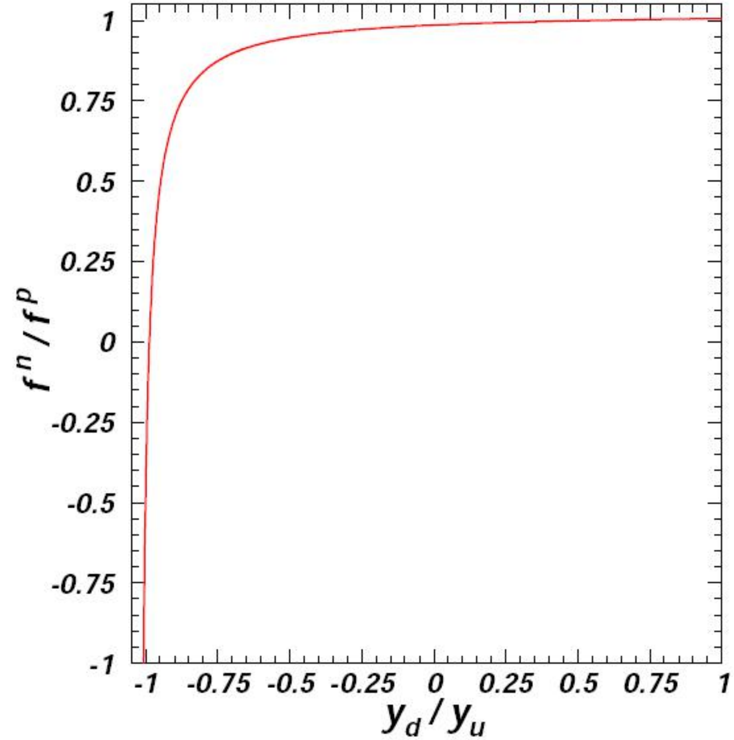
Z=54

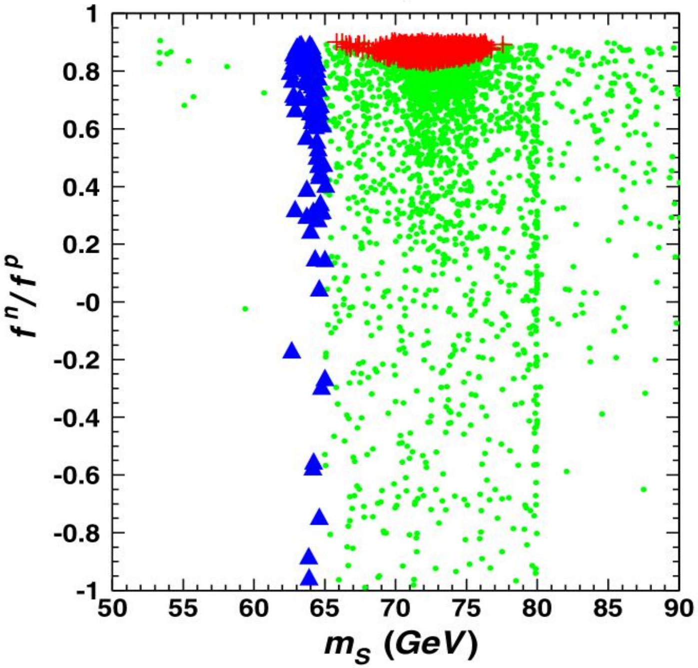
Xe	Ge	Si	Ca	W	Ne	C
128 (1.9)	70 (21)	28 (92)	40 (97)	182 (27)	20 (91)	12 (99)
129 (26)	72 (28)	29 (4.7)	44 (2.1)	183 (14)	22 (9.3)	13 (1.1)
130 (4.1)	73 (7.7)	30 (3.1)		184 (31)		
131 (21)	74 (36)			186 (28)		
132 (27)	76 (7.4)					
134 (10)						
136 (8.9)						

σ_N^Z **is the DM-nucleon cross section from scattering off nuclei with atomic number *Z* assuming isospin conservation.**

J. L. Feng, J. Kumar, D. Marfatia, D. Sanford, Phys. Lett. B 703, 124-127 (2011).







$$hd\bar{d}: \quad \frac{m_d}{v} y_d \quad (\text{d, s, b})$$

$$hu\bar{u}: \quad \frac{m_u}{v} y_u \quad (\text{u, c, t})$$

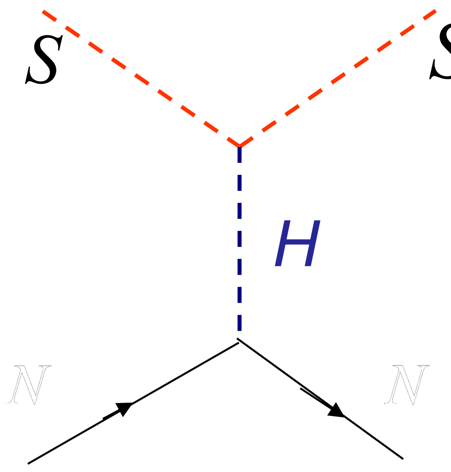
Because of the tension between Br(h->SS) and relic density, M_{DM} < 50 GeV is excluded

L W, R. Shi, X.-F. Han, PRD96 (2017) 115025

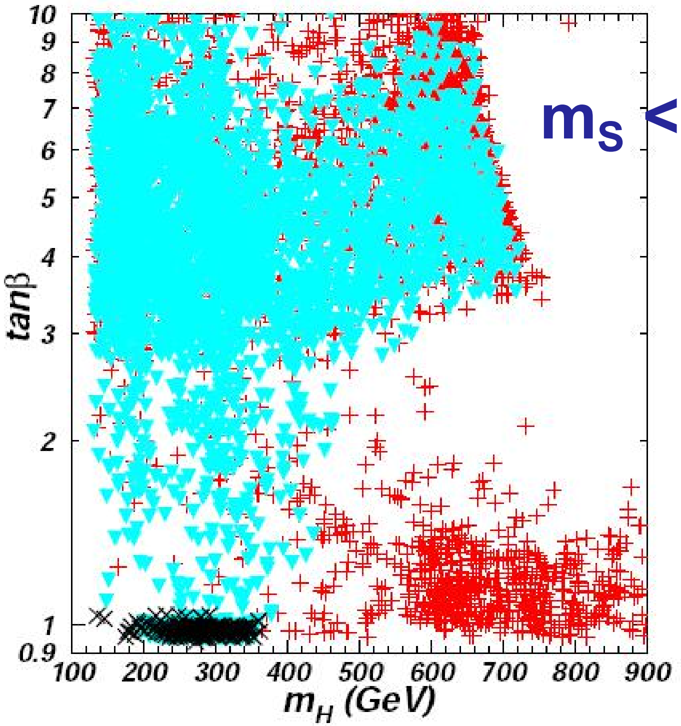
For sin(β-α)⟶1 and tanβ⟶1, if *H* is the portal between the DM and SM sectors, the isospin-violating DM interactions with nucleon can be also obtained.

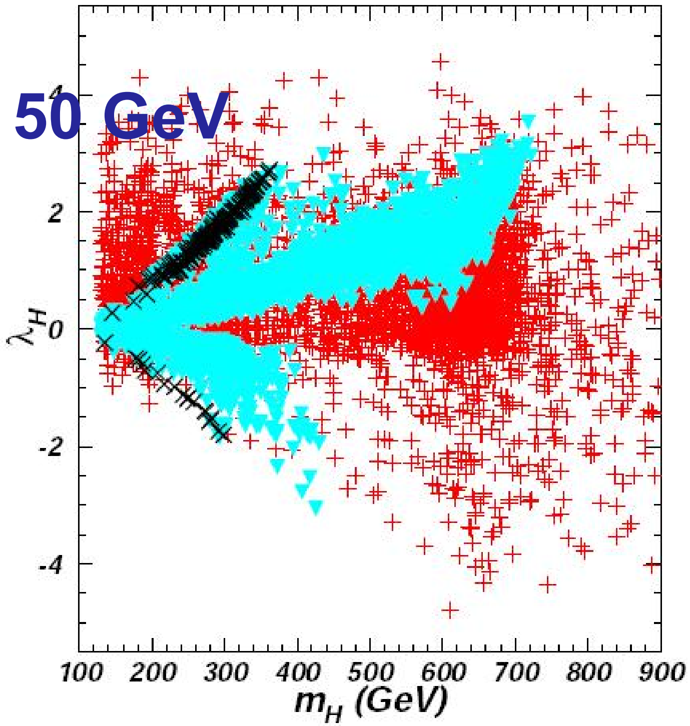
$$H\bar{u}u:\quad \frac{m_u}{v}\left[\cos(\beta-\alpha)-\sin(\beta-\alpha)\frac{1}{t_\beta}\right]$$

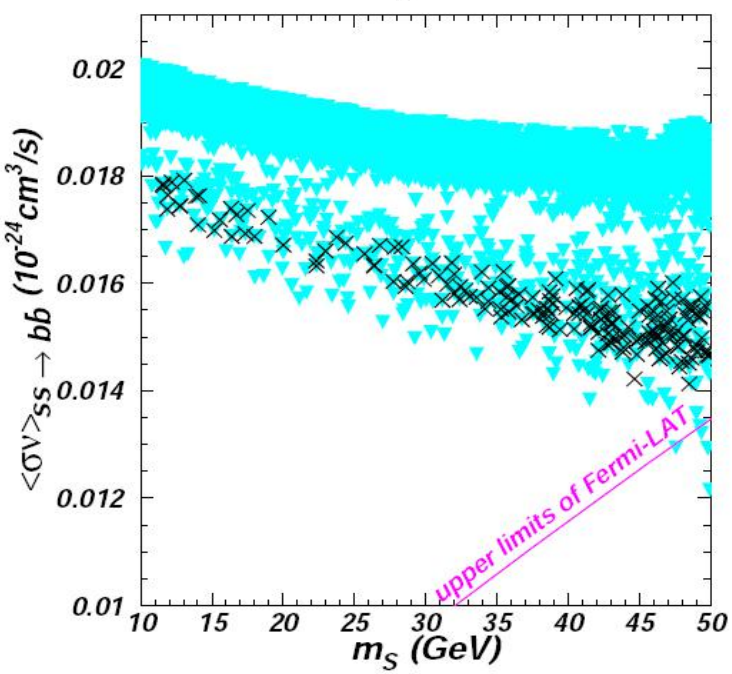
$$H\bar{d}d:\quad \frac{m_d}{v}\left[\cos(\beta-\alpha)+\sin(\beta-\alpha)\ t_\beta\right]$$



m_S < 50 GeV







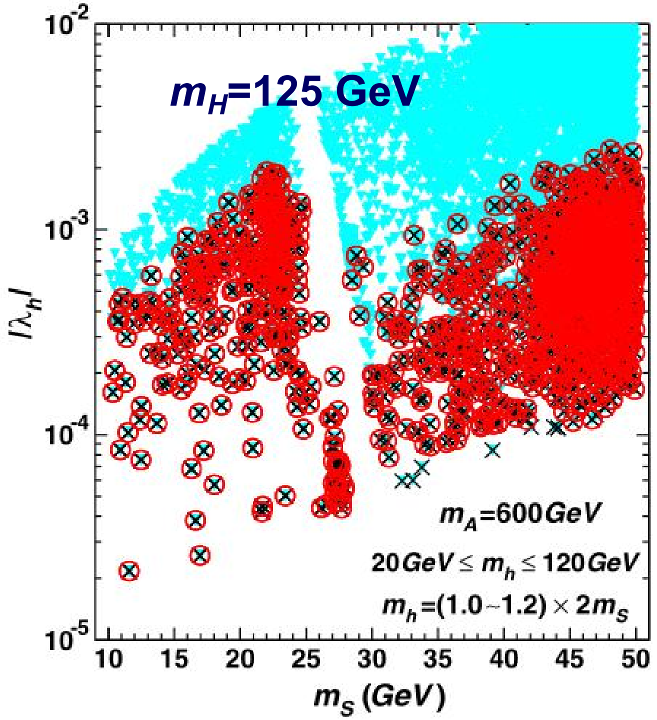
Because of the constraints of BR(h->SS) , λ_h is very small, and λ_H plays main roles.

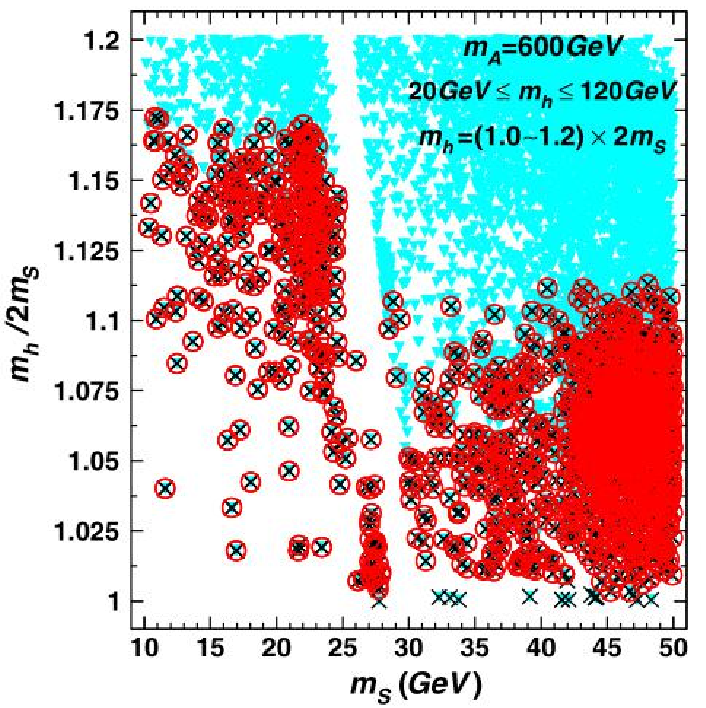
m_S < 50 GeV is excluded by Fermi-LAT searches for the DM annihilation from dwarf spheroidal satellite galaxies

Resonance effect

Non-SM-like Higgs is the portal between DM and SM sectors, m_S in the resonance region is easily allowed by the direct and indirect detection.

$$SS \to \phi \to f\bar{f},\ WW,\ ZZ$$





m_S < 50 GeV is allowed when $2m_S$ is slightly larger than m_h

Higgs-inflation in the 2HDM

Slow-roll:

From Einstein equations, we can deduce

$$\left(\frac{da/dt}{a}\right)^2 = \frac{8\pi G}{3}\rho, \qquad \frac{d^2a/dt^2}{a} = -\frac{4\pi G}{3}(\rho+3P)$$

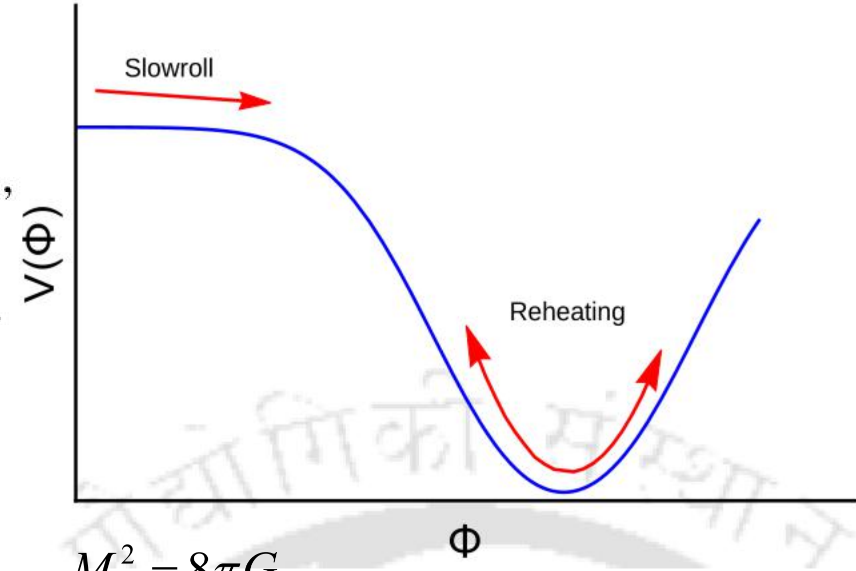
Scale factor has exponenential expansion for constant ρ and is accelerated for $P < -\frac{\rho}{3}$

For a scalar field $\mathcal{L} = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \qquad P = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

Friedmann equation: $3M_P^2\mathcal{H}^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi),$

kinetic equation: $\ddot{\phi} + 3\mathcal{H}\dot{\phi} + \frac{\partial V(\phi)}{\partial\phi} = 0.$



$M_p^2 = 8\pi G$

Slow-roll condition

$$\tfrac{1}{2}\dot{\phi}^2 \ll V(\phi) \quad |\ddot{\phi}| \ll \mathcal{H}|\dot{\phi}|$$

Appyling low-roll condition, we may obtain

$$\mathcal{H}^2 \simeq V/3M_{\rm P}^2 \qquad \dot{\phi} \simeq -V'_\phi/3\mathcal{H}$$

Define slow roll parameters:

$$\varepsilon \equiv \frac{M_p^2}{2}\rho\left(\frac{V'}{V}\right)^2, \qquad \eta \equiv \frac{M_p^2 V''}{V}$$

The field value at the end of inflation is determined by ε =1.

e-folding number: $N_e = \int_{t_i}^{t_f} dt\, H = \int_{\phi_i}^{\phi_f} d\phi \frac{H}{\dot{\phi}} = \frac{1}{M_p}\int_{\phi_f}^{\phi_i} \frac{d\phi}{\sqrt{2\varepsilon}}$

Observables:

The scalar amplitude: $P_s = \frac{H^2}{8\pi^2\epsilon} = \frac{V}{24\pi^2\epsilon M_p^2}.$

The tensor to scalar ratio: $r = \frac{2H^2}{\pi^2} \Big/ \frac{H^2}{8\pi^2\epsilon} = 16\epsilon.$

Spectrum index: $n_s - 1 = \frac{d\ log P_s}{d\ log\kappa} = 2\eta - 6\epsilon$

Inflation in inert 2HDM:

J.-O. Gong, H. M. Lee, S. K. Kang, JHEP04 (2012) 128
S. Choubey, A. Kumar, JHEP11 (2017) 080

Inflation in general 2HDM:

T. Modak, K. Oda, EPJC80 (2020) 863

In the framework of type-I and type-II 2HDM, we study the inflationary dynamics,

$$\frac{\mathcal{L}_J}{\sqrt{-g}} = \frac{R}{2} + \left(\xi_1|\Phi_1|^2 + \xi_2|\Phi_2|^2\right)R - |D_\mu\Phi_1|^2 - |D_\mu\Phi_2|^2 - V\left(\Phi_1, \Phi_2\right) ,$$

R is the Ricci scalar and reduced Planck mass M_P is taken to be 1

We make the conformal transformation on the metric,

$$g^E_{\mu\nu} = g_{\mu\nu}\Omega^2 \text{ with } \Omega^2 \equiv 1 + 2\xi_1|\Phi_1|^2 + 2\xi_2|\Phi_2|^2$$

and obtain the Einstein frame action without the gauge interaction

$$\frac{\mathcal{L}_E}{\sqrt{-g_E}} = \frac{R_E}{2} - \frac{3}{4}\left[\partial_\mu \log\left(1 + 2\xi_1|\Phi_1|^2 + 2\xi_2|\Phi_2|^2\right)\right]^2 - \frac{|\partial_\mu\Phi_1|^2 + |\partial_\mu\Phi_2|^2}{1 + 2\xi_1|\Phi_1|^2 + 2\xi_2|\Phi_2|^2} - V_E(\Phi_1, \Phi_2) ,$$

$$V_E(\Phi_1,\Phi_2) = \frac{V}{\left(1+2\xi_1|\Phi_1|^2+2\xi_2|\Phi_2|^2\right)^2}$$

We take the Higgs doublets,

$$\Phi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ h_1 \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ h_2 e^{i\theta} \end{pmatrix}$$

and ignore the mass term

$$\frac{\mathcal{L}_E}{\sqrt{-g_E}} = \frac{R_E}{2} - \frac{1}{2}S_{ij}\partial_\mu\phi^i\partial^\mu\phi^j - V_E(\phi^i)$$

$$S_{ij} = \frac{1}{1+\xi_1 h_1^2+\xi_2 h_2^2}\begin{pmatrix} 1+\dfrac{6\xi_1^2 h_1^2}{1+\xi_1 h_1^2+\xi_2 h_2^2} & \dfrac{6\xi_1\xi_2 h_1 h_2}{1+\xi_1 h_1^2+\xi_2 h_2^2} & 0 \\ \dfrac{6\xi_1\xi_2 h_1 h_2}{1+\xi_1 h_1^2+\xi_2 h_2^2} & 1+\dfrac{6\xi_2^2 h_2^2}{1+\xi_1 h_1^2+\xi_2 h_2^2} & 0 \\ 0 & 0 & h_2^2 \end{pmatrix},$$

$$V_E(\phi^i) = \frac{\lambda_1 h_1^4+\lambda_2 h_2^4+2(\lambda_3+\lambda_4)h_1^2 h_2^2+2\lambda_5 h_1^2 h_2^2\cos(2\theta)}{8\left(1+\xi_1 h_1^2+\xi_2 h_2^2\right)^2}.$$

To obtain a diagonal kinetic form, we redefine the scalar fields

$$\varphi = \sqrt{\frac{3}{2}}\log(1+\xi_1 h_1^2+\xi_2 h_2^2)$$

$$\rho = \frac{h_2}{h_1}.$$

J.-O. Gong, H. M. Lee, S. K. Kang, JHEP04 (2012) 128

Thus, the potential becomes

$$V_E(\varphi,\rho,\theta) = \frac{\lambda_1 + \lambda_2\rho^4 + 2(\lambda_3+\lambda_4)\rho^2 + 2\lambda_5\rho^2\cos(2\theta)}{8\left(\xi_1+\xi_2\rho^2\right)^2}\left(1-e^{-2\varphi/\sqrt{6}}\right)^2$$

After stabilizing θ at the minimum of potential, we obtain θ independent part of potential

$$V_{\theta\text{-indep}} \approx \frac{\lambda_1 + \lambda_2\rho^4 + 2\lambda_L\rho^2}{8\left(\xi_1+\xi_2\rho^2\right)^2}\left(1-e^{-2\varphi/\sqrt{6}}\right)^2$$

$$\lambda_L \equiv \lambda_3 + \lambda_4 - |\lambda_5|$$

The φ field can drive inflation, we discuss two simple scenario:

(1) h_2-inflation for $\rho = \infty$. The potential has extrema at ρ=∞ for

$$x_1 \equiv \lambda_2\xi_1 - \lambda_L\xi_2 < 0, \quad x_2 \equiv \lambda_1\xi_2 - \lambda_L\xi_1 > 0.$$

The potential becomes $V = \dfrac{\lambda_2}{8\xi_2^2}\left(1-e^{-2\varphi/\sqrt{6}}\right)^2$

(2) h_1-inflation for $\rho = 0$ The potential has extrema at ρ=0 for

$$x_1 > 0, \quad x_2 < 0.$$

The potential becomes $V = \dfrac{\lambda_1}{8\xi_1^2}\left(1-e^{-2\varphi/\sqrt{6}}\right)^2$

The value of φ_e at the end of inflation is determined by $\varepsilon = 1$.

$$\epsilon(\varphi) = \frac{1}{2}\left(\frac{dV(\varphi)/d\varphi}{V(\varphi)}\right)^2 , \qquad \eta(\varphi) = \frac{d^2V(\varphi)/d\varphi^2}{V(\varphi)}$$

Taking N_e=60, the horizon exit value φ_* can be calculated

$$N = \int_{\varphi_e}^{\varphi_*} d\varphi \frac{1}{\sqrt{2\epsilon}}$$

Taking N_e=60, the horizon exit value φ_* can be calculated. This allows us to calculate

$$n_s = 1 + 2\,\eta - 6\,\epsilon = 0.9678\,,$$

$$r = 16\,\epsilon = 0.003\,,$$

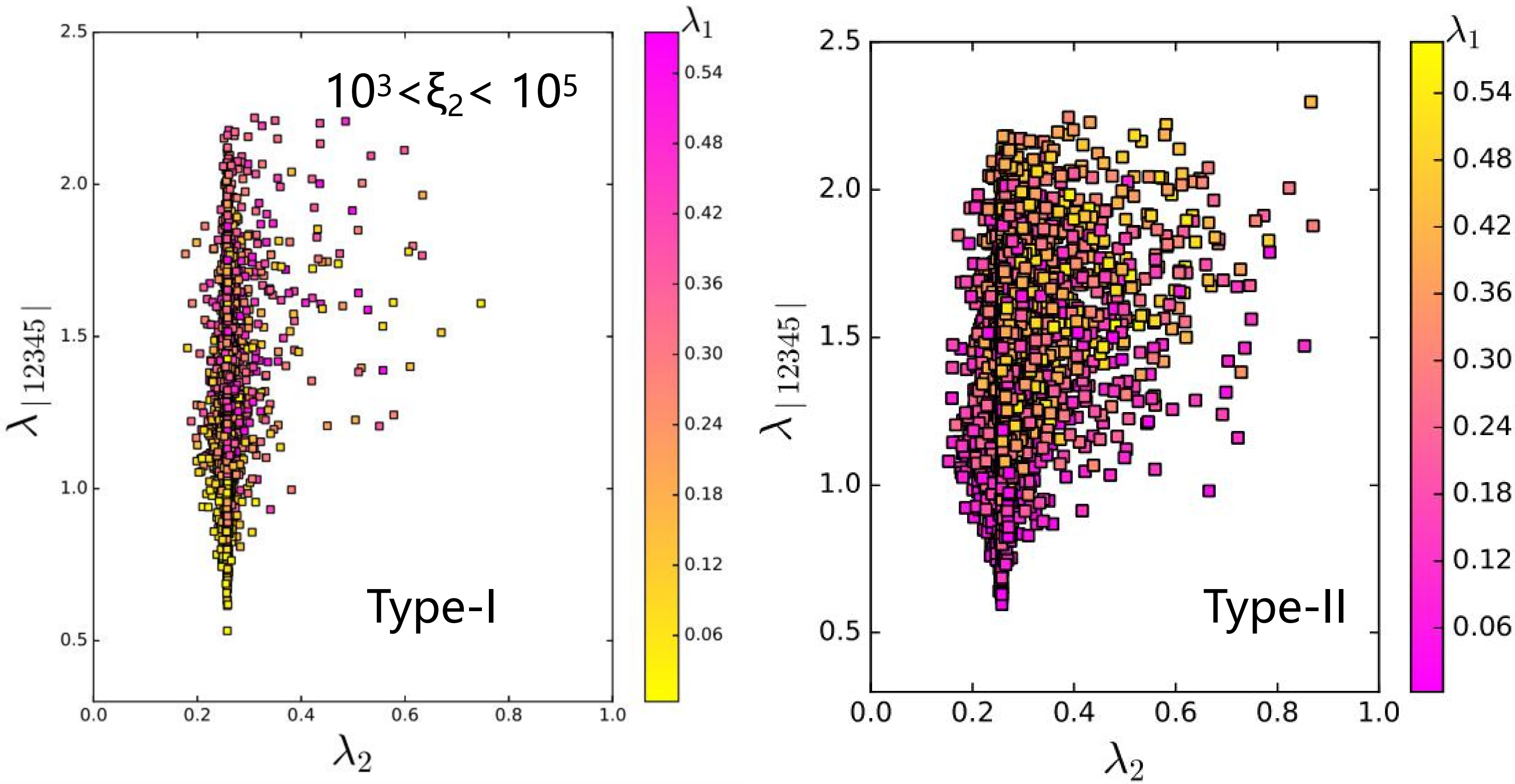
$$P_s = \frac{V}{24\pi^2\epsilon}\,.$$

The Planck collaboration reported bounds

$$n_s = 0.9649 \pm 0.0042,$$

$$r < 0.056,$$

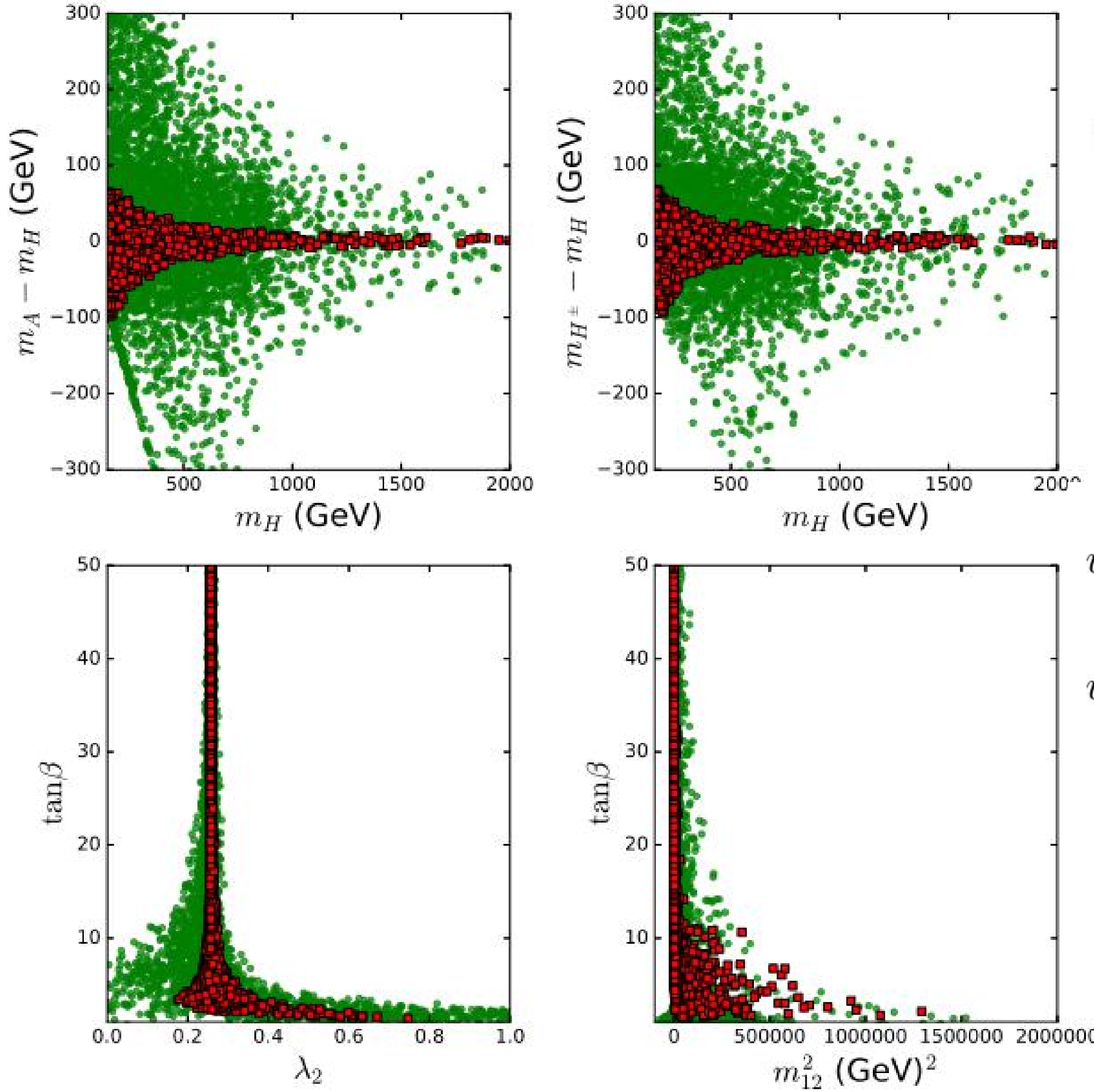
$$P_s = (2.099 \pm 0.014) \times 10^{-9}.$$



$$\lambda_{|12345|} = |\lambda_1| + |\lambda_2| + |\lambda_3| + |\lambda_4| + |\lambda_5|$$

Imposing theoretical constraints from EW scale to M_p/ξ_2 , oblique parameters, Higg signal data of 125 GeV Higgs, and condition of h_2-inflation

Type-I for h_2-inflation



Small splitting masses

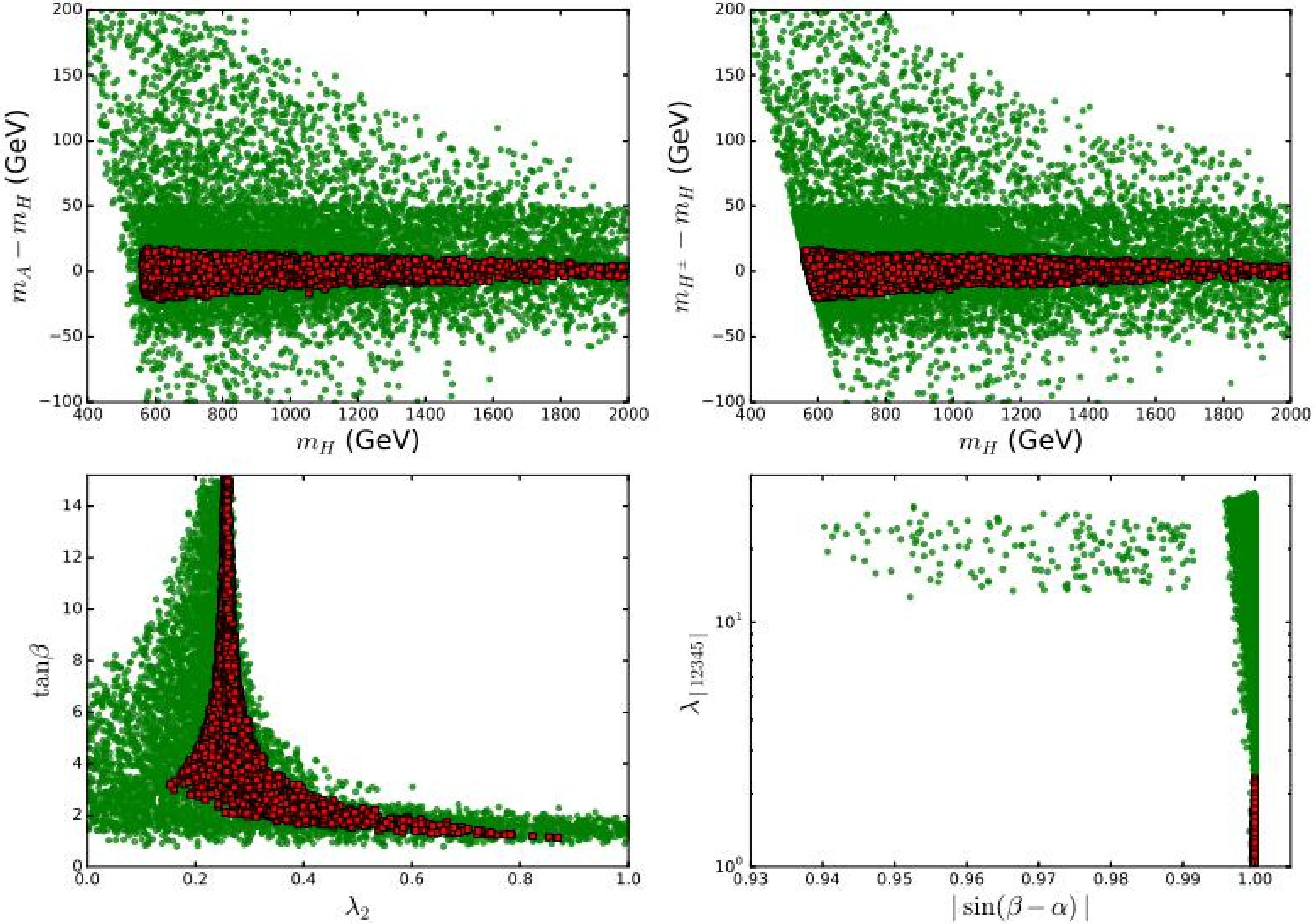
$$m_\phi^2 \approx y_\phi M^2 + f_\phi(\lambda_i) v^2 + \mathcal{O}(\frac{v^4}{M^2}),$$

$$v^2\lambda_1 = m_h^2 - \frac{t_\beta^3\,(m_{12}^2 - m_H^2 s_\beta c_\beta)}{s_\beta^2},$$

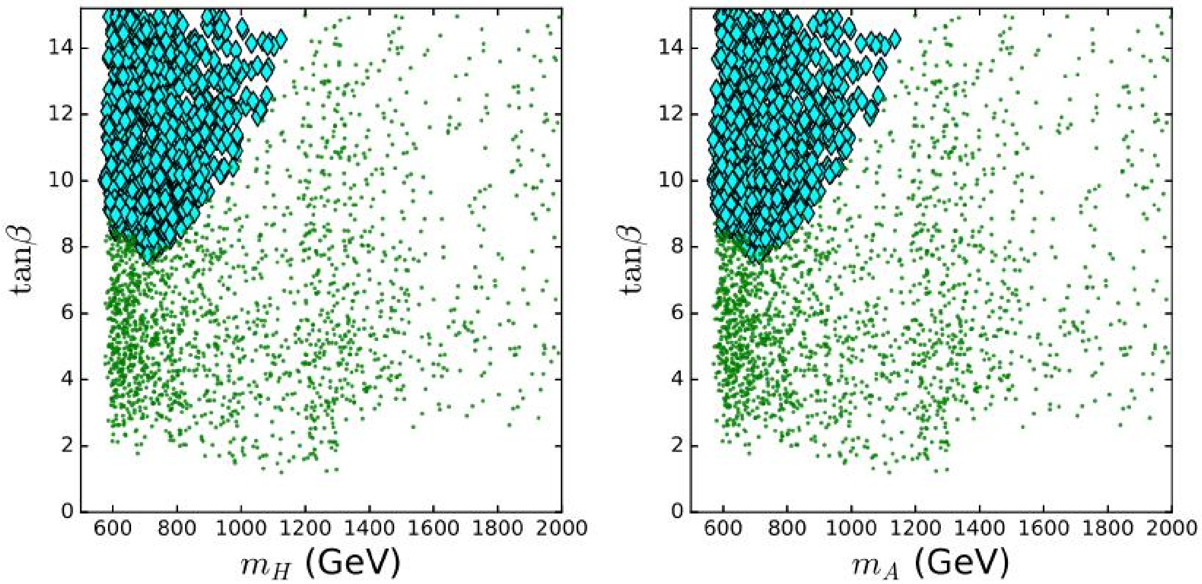
$$v^2\lambda_2 = m_h^2 - \frac{(m_{12}^2 - m_H^2 s_\beta c_\beta)}{t_\beta s_\beta^2},$$

Bullets (green) are imposed by theoretical constraints at EW

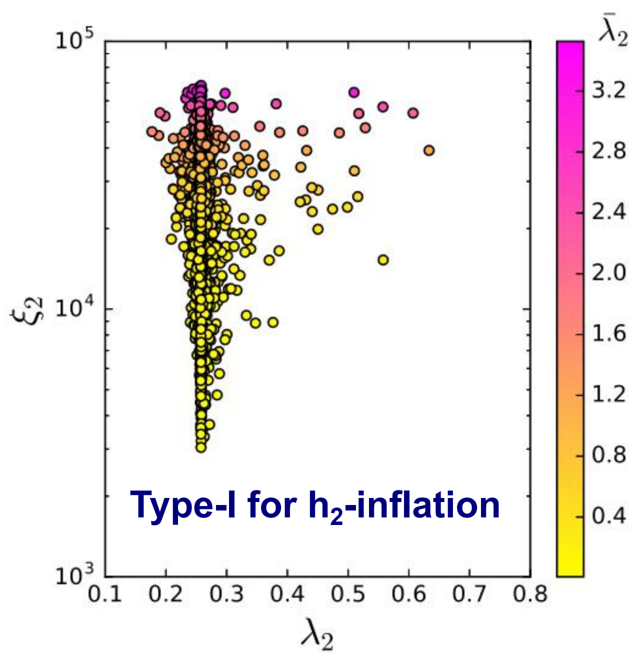
Type-II for h_2 inflation

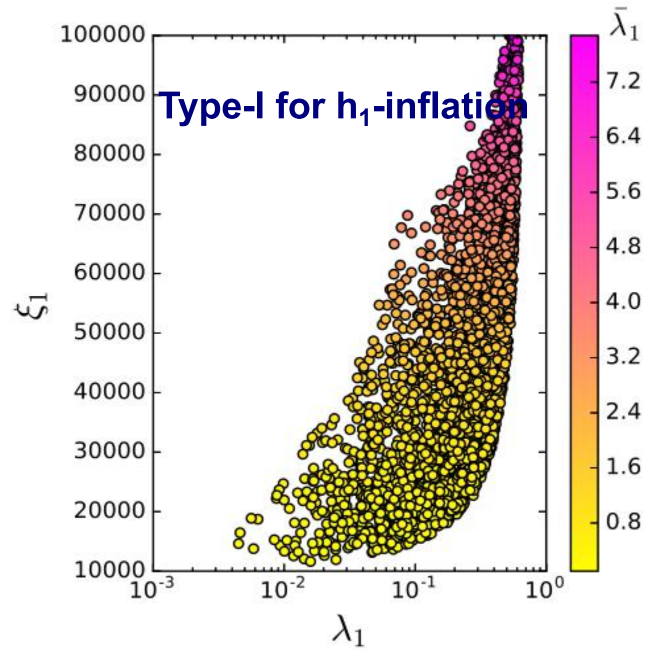


$b \rightarrow s\gamma$ requires $m_{H^\pm} > 570$ GeV in the type-II model.



Constraints of Direct searches at the LHC on Type-II for h_2-inflation

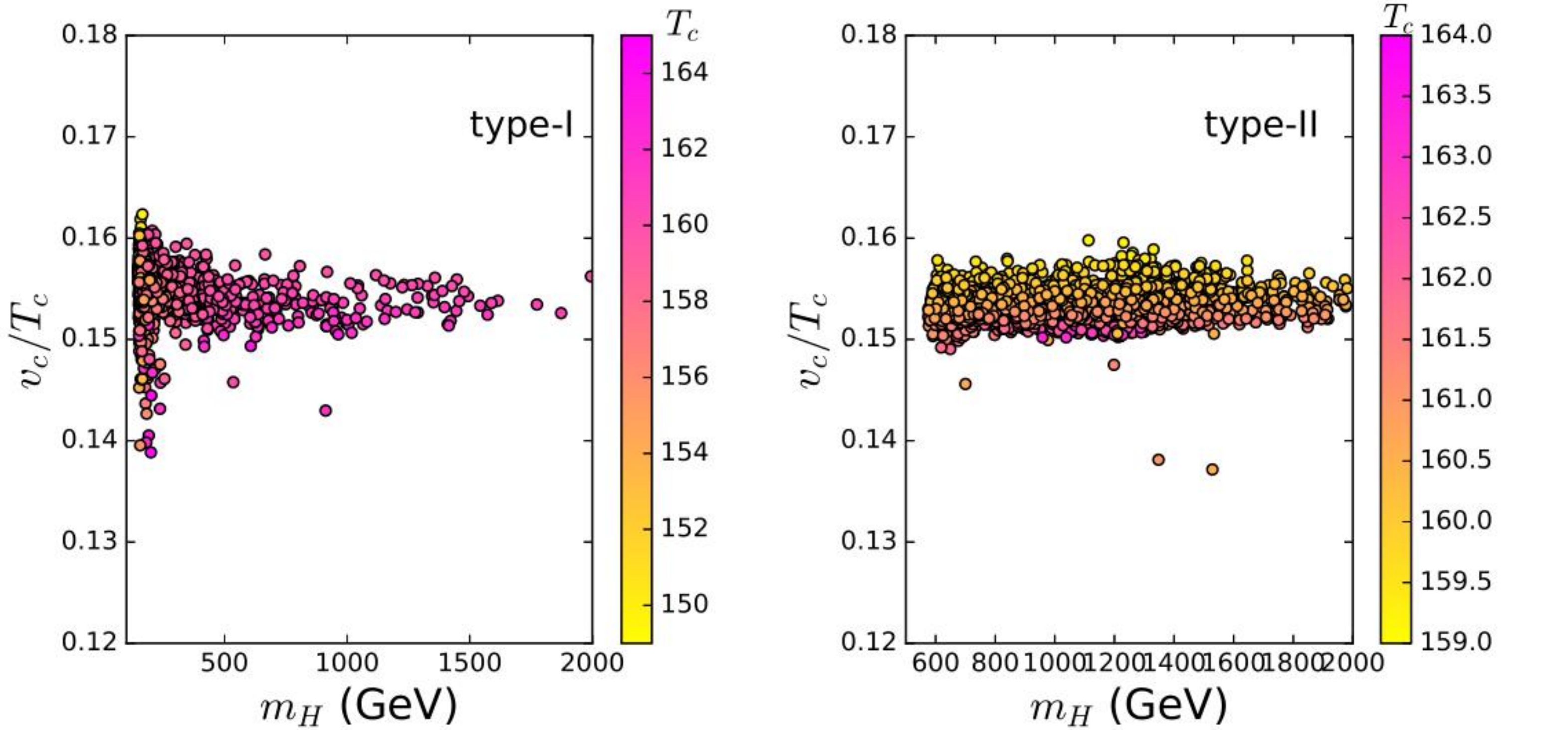




$$P_s = (2.099 \pm 0.014) \times 10^{-9}.$$

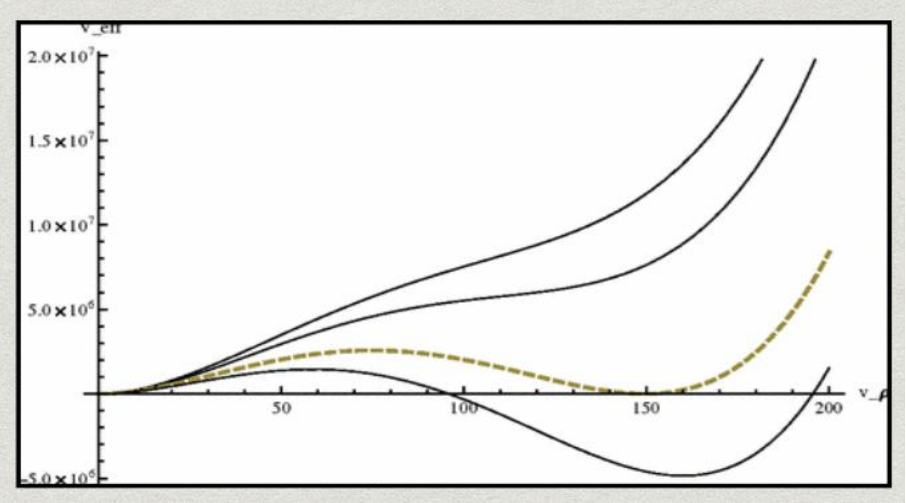
$$P_s = \frac{V}{24\pi^2 \epsilon}.$$

$$V = \frac{\bar{\lambda}_1}{8\xi_1^2}\left(1 - e^{-2\varphi/\sqrt{6}}\right)^2$$



The first order EWPT can be produced and relatively weak in the region achieving Higgs-inflation

Electroweak phase transition



The effective potential:

$$V_{\rm eff}(h_1,h_2,T) = V_0(h_1,h_2) + V_{\rm CW}(h_1,h_2) + V_{\rm CT}(h_1,h_2) + V_{\rm th}(h_1,h_2,T) + V_{\rm daisy}(h_1,h_2,T)$$

$$V_{\rm th}(h_1,h_2,T) = \frac{T^4}{2\pi^2}\sum_i n_i J_{B,F}\left(\frac{m_i^2(h_1,h_2)}{T^2}\right)$$

High temperture expansion:

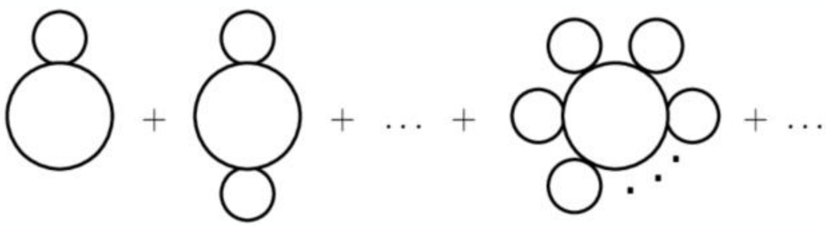
$$J_B^{y\ll 1}(y) \simeq -\frac{\pi^4}{45} + \frac{\pi^2}{12}y - \frac{\pi}{6}y^{3/2} - \frac{y^2}{32}\ln\frac{y}{a_B},$$

$$J_F^{y\ll 1}(y) \simeq -\frac{7\pi^4}{360} + \frac{\pi^2}{24}y + \frac{y^2}{32}\ln\frac{y}{a_F},$$

$$V_{\text{daisy}}(h_1,h_2,T) = -\frac{T}{12\pi}\sum_i n_i\left[\left(M_i^2(h_1,h_2,T)\right)^{\frac{3}{2}} - \left(m_i^2(h_1,h_2)\right)^{\frac{3}{2}}\right]$$

$$M_i^2(h_1,h_2,T) = \text{eigenvalues}\left[\widehat{\mathcal{M}_X^2}(h_1,h_2) + \Pi_X(T)\right]$$

The correction from the resummation of daisy diagrams

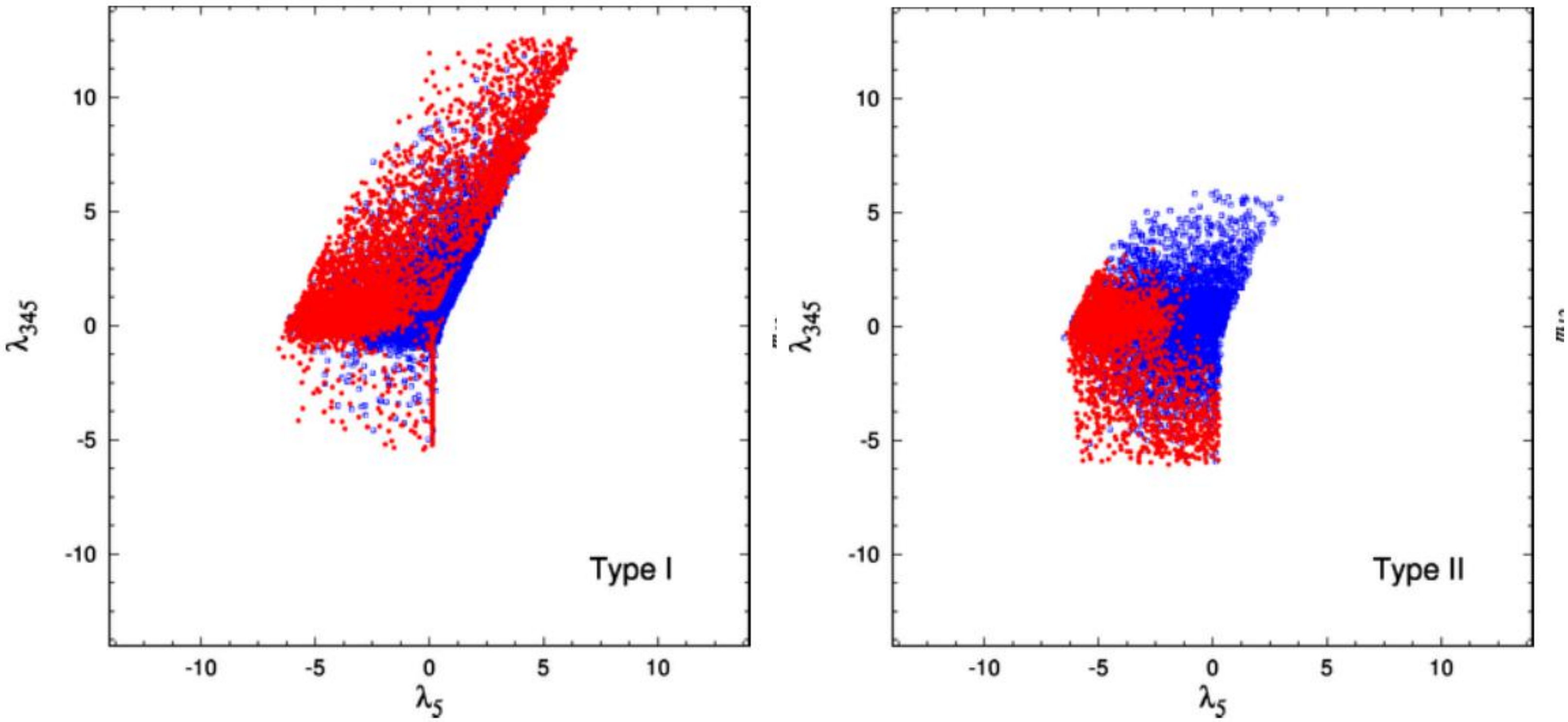


$$f_{(n)}(T)_{\text{daisy}} = \frac{(-1)^{n+1}}{n}\left(\frac{\lambda T^2}{4}\right)^n \frac{T}{2}\left[\int \frac{d^3\vec{p}}{(2\pi)^3}\frac{1}{(\vec{p}^2+m^2)^n}\right] = -\frac{T}{2}\frac{1}{n!}\left(\frac{\lambda T^2}{4}\right)^n\left(\frac{d}{dm^2}\right)^n\left(\frac{m^3}{6\pi}\right)$$

Summation:

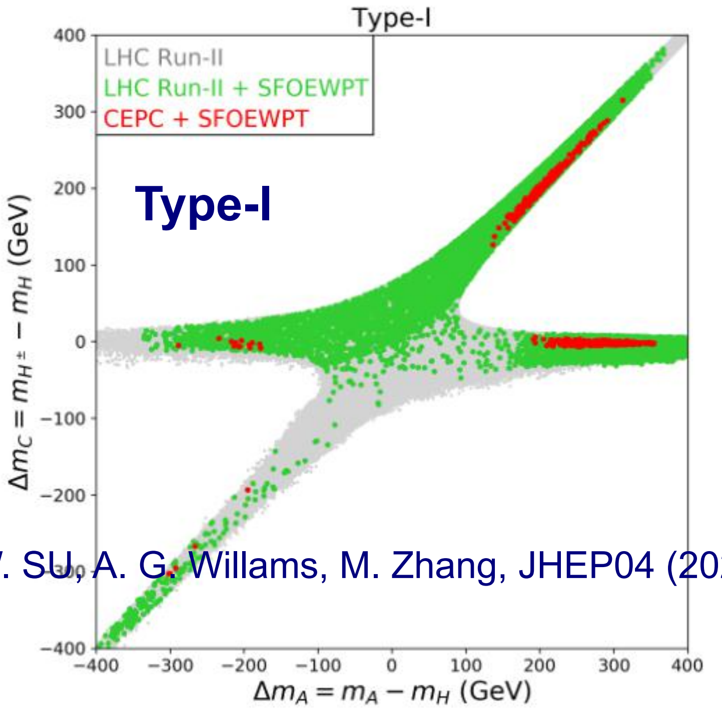
$$f_{\text{daisy}}(T) = \frac{T}{12\pi}m^3 - \frac{T}{12\pi}\sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{\lambda T^2}{4}\right)^n\left(\frac{d}{dm^2}\right)^n(m^3) = \frac{T}{12\pi}m^3 - \frac{T}{12\pi}\left(m^2+\frac{\lambda T^2}{4}\right)^{\frac{3}{2}}$$

M. Laine, A. Vuorinen, arXiv:1701.01554

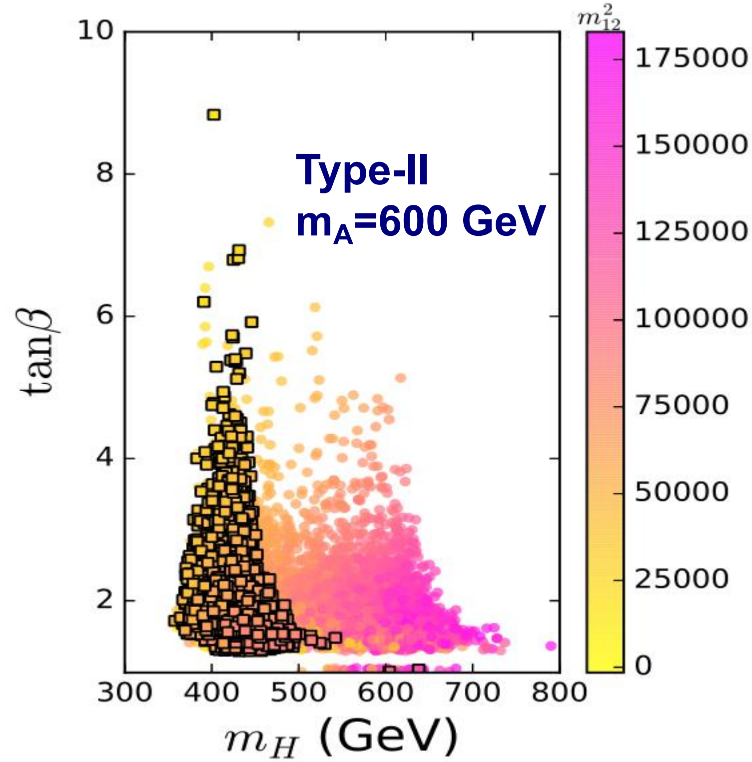


The first order (red) and second order (blue) PT

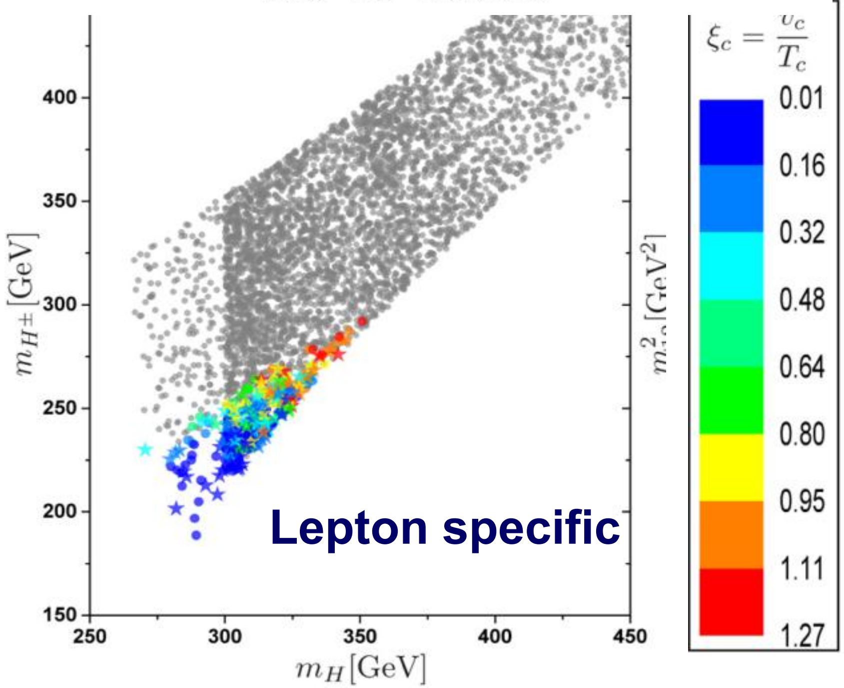
J. Bernon, L. Bian, Y. Jiang, JHEP05 (2018) 151



W. SU, A. G. Willams, M. Zhang, JHEP04 (2021) 219



X.-F. Han, L W, Y. Zhang, PRD103 (2021) 035012



A strong first order EWPT favors large mass splitting among H, A, $H^\pm$

L W, j. M. Yang, M. Zhang, Y. Zhang, PLB788 (2019) 519

Conclusions：

2HDM is a simple extention of SM, which has wide applications in many fields of the elementary particle and cosmology.

Thanks！